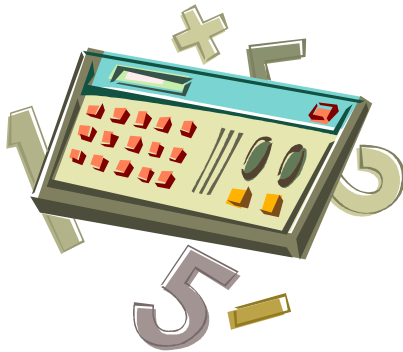


# International School of Dakar

April 2005



## MATHEMATICS CURRICULUM FRAMEWORK

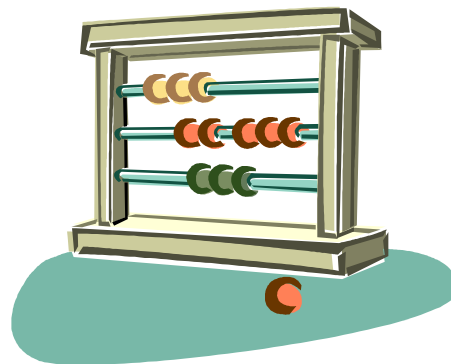
### Grades PK-12

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## Statement of Philosophy

The International School of Dakar Mathematics Curriculum is grounded in the philosophy of learning described below.

### **There is a mathematician within each of us.**

- This mathematician can be accessed
- Students should experience mathematics in a way that is both understandable and challenging
- All students should learn to recognize, respect, and nurture the mathematician within them

### **Experience with models for math concepts helps us understand, invent, and remember important math ideas.**

- Through the use of manipulatives, models, sketches and diagrams, students develop an understanding of mathematical concepts and processes and create meaningful mental images that help them retain and recall information

### **Learning mathematics is a social activity.**

- The process of examining and discussing our thoughts can reveal contradictions, insights, inconsistencies, and connections that might otherwise go unnoticed
- We learn when we question, explain, and listen to ideas
- Math discussions can be fascinating and instructive and reveal the variety of ways in which a mathematical situation can be approached and broaden the understanding of those engaged in discussion

### **Learning mathematics is an ongoing process of knowledge construction.**

- We construct new understanding each time we consider an idea from a different point of view
- The cyclical structure of the curriculum, in which ideas resurface regularly in new contexts, is based on a constructivist view of learning
- The importance of continually looking for new perspectives is emphasized along with “mastery” of any one idea and memorization of procedures

### **“Disequilibrium” stimulates new learning.**

- We develop deep and broad knowledge when we allow time for confusion to be processed
- Concrete experiences, social interactions, and reflection provide the student with perspectives that may contradict their present points of view
- Learning occurs as a result of the students’ efforts to reformulate their conceptions

- It is important that students view their struggles with problems as evidence that learning is “in progress”

### **Mathematics is a fascinating world of its own**

- The world of mathematics provides a rich and engaging context for developing an understanding of what it means to think mathematically – to grapple with difficult problems, to question and challenge ideas, and to conjecture and generalize about relationships
- As students explore the world of mathematics, they build confidence in themselves and develop a sense of the power and beauty of mathematics



## **A Vision for School Mathematics**

Imagine a classroom, school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work

and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment.

Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.

~ ~ From Chapter 1 of *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, 2000. p. 3.

## RESEARCH

### History of Mathematics Education

Mathematics has always claimed a position at the core of education as one of the “three Rs.” In the 1960’s, during the Sputnik era, “new math” was born. The attempts at “new math” failed to have an immediate effect on student learning. However, over the next 40 years many of the ideas and procedures from that reform began to be used in our classrooms. During the 1980’s an increased focus on problem solving and the introduction of calculators placed a new focus on mathematics performance.

In the 1960’s, some comparative studies were conducted (Second International Mathematics Study – SIMS) that showed U. S. students performed worse than their economic peers in other countries. In 1989, the National Council of Teachers of Mathematics (NCTM) released a set of standards that outlined goals and expectations for curriculum. The NCTM standards were based on research about learning and teaching and on effective teaching practices. In 2000, NCTM released a new document, *Principles and Standards for School Mathematics*. The document is intended to be a resource and guide for all who make decisions that affect the mathematics education of students, PK-12.

The NCTM standards reflect a vision of student empowerment in number, measurement, geometry, data and chance, and algebra, but many discussions about the school curriculum continue to be centered on basic facts of curriculum and performance of rote algorithmic skills in both numeric and algebraic settings. “The public, in general, is skeptical of letting go of the past and looking forward to classrooms where students engage in mathematics in exploratory settings. Such settings call on students to use their understanding of the necessary mathematical concepts and skills to solve problems.”

Acknowledgements:

~ *Mathematics, A Chapter of the Curriculum Handbook*, ASCD, 1999, [www.ascd.org](http://www.ascd.org).

~ *Principles and Standards for School Mathematics*, NCTM, Reston, VA, 2000, [www.nctm.org](http://www.nctm.org).

### What are the issues facing PK-12 Mathematics Today?

#### How are our students really doing?

ITBS: Each year student from grades 3-8 take the Iowa Test of Basic Skills (ITBS). The ITBS is a norm-referenced test (NRT). The main reason for using norm-referenced tests is to classify students. Norm-referenced tests are designed to highlight achievement difference between and among students to produce a dependable rank order of students across a continuum of achievement from high achievers to low achievers (Stiggins, 1994). With norm-referenced tests, a representative group of students is given the test prior to its availability to the public. The scores of students who take the test after publication are then compared to those of the norm group. The Iowa Test of Basic Skills is normed using a national and international group of students. A summary of ITBS results for International School of Dakar over the past 5 years is located in the addendum portion of this

curriculum. The scores are reported by class group so that the reader can see the progress of a group of students over time. The reader will note that both national and international norms are displayed. The national score compares ISD students to the norm group in the United States, and the International score compares ISD students to a norm group outside of the United States.

NAEP and TIMSS Results: Since ISD follows a U.S. curriculum, it was felt that information related to the general achievement of students in the United States was important to share. The committee when writing this curriculum considered research from NAEP and TIMSS. The National Assessment of Educational Progress (NAEP) presents information on student performance over time, and the Third International Mathematics and Science Study (TIMSS) provides data comparing U.S. student performance with that of their international peers. The research conducted by NAEP and TIMSS can be found in the addendum section of this curriculum.

### **What is school mathematics?**

The setting of curriculum standards by the National Council of Teachers of Mathematics from 1989-1995 led to the first steps of reform. The current standards frame the goals for mathematics education around six principles.

1. Equity Principle – promotes the learning of mathematics by all students
2. Mathematics Curriculum Principle – promotes curriculums that are thorough and comprehensive
3. Teaching Principle – depends upon competent, caring teachers who teach all students to understand and use mathematics
4. Learning Principle – enable students to understand and use math
5. Assessment Principle – include assessments to monitor, enhance, and evaluate the mathematics learning of all student to inform teaching
6. Technology Principle – use technology to help all students understand mathematics and to prepare them to use mathematics in an increasingly technological world

According to the NCTM there are 10 general standards central to the development of a good mathematics program. Five deal with content, and five deal with processes.

#### **Content**

- Number and Operation
- Patterns, Functions, and Algebra
- Geometry and Spatial Sense
- Measurement
- Data Analysis, Statistics and Probability

#### **Processes**

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

### **How should instruction be conducted?**

At the center of many of the changes associated with mathematics education is the instruction that takes place in the classroom. The recommendations for change call for the use of a broad range of instructional approaches. Instruction that promotes lasting and integrated learning is the most meaningful. The view of learning that calls on student involvement in making sense out of situations is often classified under the general rubric of constructivism. Approaches that value student construction of meaning include:

- cooperative learning groups
- essential questions
- student projects
- use of manipulatives
- peer teaching
- communicating through writing
- reflective journals

**The big difference in the constructivist approach and a more traditional approach is that the teacher is not the “teller” but the “shaper” of student knowledge.**

Teachers need to understand that what students know depends to a great extent on how they learned it. Mathematics is not memorization, even though there are things like the basic facts of arithmetic that must be developed to the point of recall. However, that recall should be based on knowledge of why a given fact has the value that it does.

### **How should students be assessed?**

“The language about student assessment has stressed the importance of “performance” and “authentic” assessments. These are assessments that measure students’ knowledge in a way that makes sense to the students and allows them to communicate in their own fashion exactly what they know and do not know in a given topic.” (ASCD Curriculum Handbook, 1999)

The indiscriminate use of standardized testing in schools has resulted in an overemphasis on basic skills and algorithms. To counteract these results, schools must increase the variety and weight assigned to forms of assessment. By shifting the weight assigned to various assessments, the reporting of student growth can move toward the goals of the standards.

The first step is to get teachers to recognize the differences between collecting information for the purpose of improving student performance, and changing their teaching (i.e., assessment) and for the purpose of ranking and grading students (i.e., evaluation). These two purposes have historically been skewed in the direction of evaluation. Assessment measures such as rubrics, journals, performance assessments, and extended projects can be used to reinforce the “processes” of learning mathematics as well as the facts, concepts, and skills.

Norm-referenced tests, such as the Iowa Test of Basic Skills, have come under attack recently because they traditionally have seemed focused on low level, basic skills. This emphasis is in direct contrast to the recommendations made by the latest research on teaching and learning which calls for educators to stress the acquisition of conceptual understanding as well as the application of skills. The National Council of Teachers of Mathematics has been particularly vocal about this concern. In an NCTM publication (1991), Romberg cited that “a recent study of the six most commonly used commercial achievement tests found that at grade 8, on average, only 23 percent of the items were problem solving while 77 percent were computation or estimation.”

# Mathematics Textbook Adoption

2004-2005 School Year

## Grades K-5, Elementary

*Harcourt Math*: ©2004

Harcourt Education  
Orlando, FL 32887  
www.harcourtschool.com

## Grades 6-8, Middle School

*Middle School Math, Course 1*, Grade 6: ©2004

*Middle School Math, Course 2*, Grade 7: ©2004

*Middle School Math, Course 3*, Grade 8: ©2004

**Holt, Rinehart and Winston**  
10801 N. MoPac Expy., Building 3  
Austin, TX 78759-5415  
www.hrw.com

## Grades 9-12, High School

### **ALGEBRA I**

*Algebra 1* by Larson, Boswell, Kanold and Stiff: ©2004

### **ALGEBRA 2**

*Algebra 2* by Larson, Boswell, Kanold and Stiff: ©2004

### **GEOMETRY**

*Geometry* by Larson, Boswell, Kanold and Stiff: ©2004

**McDougal Littell**  
P.O. Box 1667  
Evanston, IL 60204  
www.mcdougallittell.com

### **PRECALCULUS**

*Precalculus*, Sixth Edition, M. Sullivan ©2002

**Prentice Hall School Division**  
1 Lake Street, Upper Saddle River, NJ 07458  
www.PHSchool.com

### **CALCULUS AND AP CALCULUS**

*Calculus: Graphical, Numerical, Algebraic* by Finney, Demana, Waits and Kennedy: ©1999

**Addison Wesley**  
75 Arlington Street  
Suite 300  
Boston, MA 02116  
[www.aw-bc.com/home](http://www.aw-bc.com/home)

## Concepts and Procedures of Mathematics

- **Number Sense & Computation:** Numbers are everywhere and enter students' lives in many contexts. Children begin school with a knowledge of numbers gathered from their life experiences and their use of physical objects. Students who are developing number sense are actively involved in personal experiences in constructing numbers, understanding how numbers relate to each other, providing reasonable answers to questions about numbers and using numbers effectively in many different situations. Developing number sense is a lifelong process.
- **Measurement:** Students naturally compare attributes of objects in their lives, intuitively identifying which one is larger or heavier or longer. At school, students need many experiences over time to confront their beliefs about conservation (for example, is there really more clay when the ball is flattened?). Students also need many chances to construct their own understandings about measuring devices and standard units of measure by comparing two things that are not in the same room or communicating their findings to other people.
- **Geometry & Spatial Sense:** Long before they can use the language of geometry, students become aware of shape. Before entering school, they have had many experiences with points, lines, planes, and spaces. In school, students need to extend that knowledge, developing spatial sense and learning to see the world through the eyes of geometry. That knowledge can come from activities that use geometry to solve problems by constructing, drawing, measuring, visualizing, comparing, describing, and transforming shapes and objects.

- **Probability:** We know with certainty that some things will happen and that some things will not. With varying degrees of likelihood, many things may or may not happen. We can help students examine these ideas by looking at daily events to see patterns and trends and by setting up experiments that allow us to analyze probabilities.

- **Statistics (Data Analysis):** Students often think that mathematics has nothing to do with real life because the problems they encounter in school are clean and neat. However, the problems in real life are complex and often do not have obvious causes or solutions, the ability to gather, represent, and analyze data in order to solve problems is increasingly important in a society based on technology and information. The study of data analysis highlights the importance of questions, conjecturing, and searching for relationships.

- **Patterns, Functions and Relationships (Algebra):**

Through the study of patterns, students learn to see relationships and make connections, generalizations, and predictions about the world around them. Looking for patterns helps bring order, cohesion, and predictability to seemingly unorganized situations. The recognition of patterns and functional relationships is a powerful problem-solving tool that enables one to simplify otherwise unmanageable tasks and to make

## **Mathematical Problem Solving**

A problem involves a new and unfamiliar situation in which a solver must find a way to attain a desired end when the way is not obvious or the end immediately attainable. The context of problem situations evolves from everyday school, home and community experiences in the early grades to increasingly complex real world and mathematical ideas that take advantage of the expanding capabilities of older students.

## **Mathematical Reasoning**

While concrete materials are appropriate for use at all levels to support logical arguments, their use in the early grades and any new situation is critical. Students should become increasingly capable of more formal reasoning and abstraction.

## **Mathematical Communication**

Clarity of thinking and depth of understanding is promoted by representing, describing, and explaining mathematical investigations and the associated concepts and processes. Interaction among students also helps them construct knowledge and learn multiple ways to think about mathematical ideas.

## **Mathematical Connections**

The power of mathematics is enhanced by the integration of mathematical content in contrast to the traditional practice of viewing mathematics as a collection of unrelated parts. Applications across the disciplines and in real world contexts provide a reason for the study of mathematics. The components: Relates Within Mathematics, Relates Among Disciplines.

# Concepts and Procedures of Mathematics

## STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

### PRE KINDERGARTEN

To meet this standard, the student will:

**Benchmark: PK.1.1 Number Sense & Computation**

**Indicators:**

- Count objects 0-10
- Identify numbers 0-10
- Rote count 0-20
- Demonstrate 1-1 correspondence to 10
- Understand the concept of number sets
- Experience quantity and number

**Benchmark: PK.1.2 Measurement**

**Indicators:**

- Compare objects by size (big, small, more, less, same)
- Explore calendar
- Begin to develop a sense of time (day and night)
- Begin to understand the sequence of days of the week
- Understand the concept of one-half
- Understand and use comparative words

**Benchmark: PK.1.3 Geometry and spatial sense**

**Indicators:**

- Recognize geometric shapes and their names (circle, square, triangle, heart, star)
- Sort and classify into groups according to shapes, color and/or size
- Explore the concept of spatial relationships
- Sort objects into subgroups that vary by one or two attributes
- Explore the concept of parts of a whole (puzzles)
- Order several objects on the basis of one attribute
- Explore the use of terms used to describe geometric attributes

**Benchmark: PK.1.4 Probability and Statistics (Data Analysis)**

**Indicators:**

- Make and interpret graphs using objects and picture
- Compare data using terms such as more/less, big/small, and ordinal position words first, second, last
- Begin to understand the concept of positional words: up, down, top, under, next to

**Benchmark: PK.1.5 Patterns, Functions & Relationships (Algebra)**

**Indicators:**

- Observe and creates patterns
- Recognize simple patterns and duplicate them

# Mathematical Problem Solving

## STANDARD 2

The student uses mathematics to define and solve problems.

### PRE KINDERGARTEN

**To meet this standard, the student will:**

**Benchmarks: PK.2.1 Investigate Situations**

**Indicators:**

- Recognizes simple patterns
- Duplicates and/or creates patterns with 2 or 3 attributes

**Benchmark: PK.2.2 Formulate Questions**

**Indicators:**

N/A

**Benchmark: PK.2.3 Construct Solutions**

**Indicators:**

N/A

# Mathematical Reasoning

## STANDARD 3

The student uses mathematical reasoning.

### PRE KINDERGARTEN

To meet this standard, the student will:

**Benchmarks:** PK.3.1 Analyzes Situations

**Indicators:**

- Compare and contrast information

**Benchmark:** PK.3.2 Predicts Results:

**Indicators:**

N/A

**Benchmark:** PK.3.3 Draws Conclusions and Verifies Results

**Indicators:**

N/A

# Mathematical Communication

## STANDARD 4

The student communicates knowledge and understanding in both everyday and mathematical language

### PRE KINDERGARTEN

To meet this standard, the student will:

**Benchmark:** PK.4.1 Gathers Information

**Indicators:**

- Use listening and observation to access and extract mathematical information such as pictures, physical models, classmates, and symbolic representations

**Benchmark: PK.4.2 Interprets Information**

**Indicators:**

- Share and explain mathematical ideas using comparative terms (more, less, big, small) and graphs

**Benchmark: PK.4.3 Represents and Shares Information**

**Indicators:**

- Express mathematical ideas orally and through graphing

## Mathematical Connections

### STANDARD 5

The student understands how mathematical ideas connect to other subject Areas and real-life situations

### PRE KINDERGARTEN

To meet this standard, the student will:

**Benchmark: PK.5.1 Relates within Mathematics**

**Indicators:**

- Begin to recognize mathematical patterns and ideas in familiar situations in other disciplines
- Begin to use mathematical thinking and modeling in familiar situations in other disciplines

**Benchmark: PK.5.2 Relates in Real-Life Situations**

**Indicators:**

- Begin to identify examples of uses of mathematical in own daily life
- Begin to identify examples of uses of mathematics in situations that relate to society

## Concepts and Procedures of Mathematics

### STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

# KINDERGARTEN

To meet this standard, the student will:

## Benchmark: K.1.1 Number Sense & Computation

### Indicator:

- Rote count up to 100
- Count objects with reasonable accuracy to 30
- Demonstrate 1-1 correspondence to 30
- Recognize written numbers to 30
- Write numbers 0-30 (some reversals expected)
- Demonstrate an understanding of more and less
- Estimate 10-20 objects
- Use vocabulary such as –shorter, longer, higher, etc.
- Instantly recognize number arrangements, or number groups to 10
- Describe parts of numbers to 10
- Recognize, name and write numerals by 10's to 100
- Recognize, name and write numerals in/out of sequence
- Identify ordinal numbers: first, second, third
- Count-on from any number
- Demonstrate understanding of number value

## Benchmark: K.1.2 Measurement

### Indicators:

- Compare objects by size, (taller, shorter, longer, same, bigger, smaller, more, less, etc.)
- Determine how long something is in non-standard measure
- Begin to understand the invariance of quantity under change of shape by having many experiences pouring liquids, sand, rice, etc.
- Begin to develop a sense of time (morning, noon, afternoon, evening, etc.)
- Identify days of the week in relation to yesterday, today and tomorrow
- Explore calendar
- Explore money in context; recognizes penny as one cent, can count pennies to ten cents
- Use math vocabulary for quantity and size (more, fewer, less, same, larger, smaller)
- Understand blocks of time (week, months)
- Understand passing of time (yesterday, today)
- Understand the fractional concept of half
- Identify instruments used to measure length (ruler), weight (scale), time (clock: digital/analog; calendar: day, month, season), temperature (thermometer)
- Name and make shapes (circle, square, triangle, oval, diamond, rectangle)
- Tell time to the hour

## Benchmark: K.1.3 Geometry & Spatial Sense

### Indicators:

- Explore two and three dimensional shapes to see how they fit together
- Begin to recognize basic shapes (spatial and plane shapes)
- Look for shapes in their world
- Explore sorting of shapes by various attributes
- Uses math vocabulary for position (above, below, beside, next to, over, under, top, bottom, middle)
- Seriate in numerical/chronological order
- Explore symmetry
- Match, combine and create sets
- Name attributes of a set
- Describe and identify sets

**Benchmark: K.1.4 Probability & Statistics Probability (Data Analysis)**

**Indicators:**

- Explore data collection
- Create and respond to a concrete graph (i.e. tally charts)
- Make and interpret bar graphs
- Explore concepts of likely and unlikely
- Investigate and describe the results of dropping a two-colored counter or using a multicolored spinner

**Benchmark: K.1.5 Patterns, Functions & Relationships (Algebra)**

**Indicators:**

- Copy and extend simple repeating patterns
- Create simple repeating patterns
- Interpret patterns verbally
- Recognize patterns in the environment
- Recognize, duplicate and extend patterns
- Add and subtract whole numbers using up to 10 concrete items
- Solve simple oral word problems
- Investigate and recognize patterns from counting by fives and tens using concrete objects

# Mathematical Problem Solving

**STANDARD 2**

**The student uses mathematics to define and solve problems.**

## KINDERGARTEN

To meet this standard, the student will:

**Benchmark: K.2.1 Investigate Situations**

**Indicators:**

- Conduct explorations
- Search for and identify patterns
- Create and expand patterns with three attributes
- Use a variety of strategies and approaches
- Recognize when information is missing or extraneous
- Recognize when an attempted approach is unproductive and tries a new approach
- Apply appropriate operations to solve problems

**Benchmark: K.2.2 Formulates Questions**

**Indicators:**

- Identify questions to be answered
- Define problems in familiar situations
- Identify the unknowns in familiar situations

**Benchmark: K.2.3 Constructs Solutions**

**Indicators:**

- Organize relevant information
- Select and use tools
- Use a variety of strategies
- Apply appropriate concepts and procedures

# Mathematical Reasoning

## STANDARD 3

The student uses mathematical reasoning.

## KINDERGARTEN

To meet this standard, the student will:

**Benchmarks: K.3.1 Analyzes Situations**

**Indicators:**

- Interpret information in relatively familiar situations
- Compare and contrast information

- Validate thinking using models, known facts, patterns and relationships

**Benchmark: K.3.2 Predicts Results:**

**Indicators:**

- Make predictions based on prior experience and known facts

**Benchmark: K.3.3 Draws Conclusions and Verifies Results**

**Indicators:**

- Test predictions by finding example to support or contradict them
- Supports arguments and justifies results based on experiences
- Check for reasonableness of results using concepts and procedures
- Reflect on and evaluate procedures and results in familiar situations

# Mathematical Communication

## STANDARD 4

The student communicates knowledge and understanding in both everyday and mathematical language

## KINDERGARTEN

To meet this standard, the student will:

**Benchmark: K.4.1 Gathers Information**

**Indicators:**

- Create a plan for collecting information
- Use reading, listening and observation skills to access and extract mathematical information from a variety of sources such as pictures, diagrams, physical models, classmates, oral narratives, and symbolic representations

**Benchmark: K.4.2 Interprets Information**

**Indicators:**

- Organize mathematical information
- Clarify mathematical understanding s in at least one way – reflecting, verbalizing or discussing
- Share, explain and defend mathematical ideas – concepts and procedures – using terms, language, charts, and graphs that can be clearly understood by a variety of audiences

**Benchmark: K.4.3 Represents and Shares Information**

**Indicators:**

- Express ideas using mathematical language and notation such as physical or pictorial models, charts and graphs
- Express mathematical ideas to teachers and peers in appropriate ways

# Mathematical Connections

**STANDARD 5**

**The student understands how mathematical ideas connect to other subject areas and real-life situations**

## KINDERGARTEN

**To meet this standard, the student will:**

**Benchmarks: K.5.1 Relates within Mathematics**

**Indicators:**

- Recognize mathematical patterns and ideas in familiar situations in other disciplines
- Use mathematical thinking and modeling in familiar situations in other disciplines

**Benchmark: K.5.2 Relates in Real-Life Situations**

**Indicators:**

- Identify examples of uses of mathematical in own daily life
- Identify examples of uses of mathematics in situations that relate to society

# Concepts and Procedures of Mathematics

## STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

## GRADE 1

To meet this standard, the student will:

**Benchmark: 1.1.1 Number Sense & Computation**

**Indicators:**

- Internalize number combinations to 10 (12 would be the target number)
- Demonstrate conservation of number to 30
- Instantly recognize a numeral to 50; writes numerals to 50
- Recognize more, and less, equal to able to tell one or two more and one or two less
- Count objects with reasonable accuracy to 100 (1 to 1 correspondence)
- Develop system for keeping track when counting objects
- Count by 1s to 100, by 2s to 100, by 5 to 100, by 10s to 100 and backward from 20
- Use fractional numbers to divide a whole into equal parts  $\frac{1}{2}$ ,  $\frac{1}{4}$ , at the concrete level to create “fair” share
- Count on and count back
- Develop strategies for computing using a variety of concrete materials
- Estimate with 30-40 objects
- Recognize and interpret symbols +, -, =
- Record equations made with manipulatives
- Act out story type problems
- Decompose numbers and form new combinations to solve addition and subtraction problems mentally
- Begin to understand fact families
- Recognize number words (1-10) as sight words
- Identify, represent, unite, compare and order sets and numbers to 18 ordinal numbers
- Master addition and subtraction facts to the total of 20 through the use of manipulative

**Benchmark: 1.1.2 Measurement**

**Indicators:**

- Estimate measurements
- Measure length, height, weight, capacity with non-standard units (paper clips, teddy bear counters, cubes, etc)
- Match and order quantities

- Develop understanding of the invariance of quantity under change of shape by having many experiences pouring liquids, sand, rice, etc.
- Explore money in context, recognizes and tells value of penny, nickel, dime
- Explore calendar
- Read hour and half hour time on standard (analog and digital) clock
- Recite days of week, month of the year and year
- Know the days of the week in order and out of order
- Experience standard measurement: length, weight, volume
- Estimate time, sequence events, use a calendar
- Identify fractional parts of an object using halves, thirds, fourths
- Find length using a ruler
- Experience liquid measures and weighing
- Identify seasons
- Select appropriate tools for measuring and estimating length, weight and temperatures (ruler, thermometer, scale)
- Compare the volumes of two given containers by using concrete materials (e.g., beans, sand, water, rice)
- Compare the weight of two objects

**Benchmark: 1.1.3 Geometry & Spatial Sense**

**Indicators:**

- Group objects by attributes using a single property: number of sides, corners, faces, straight or curved lines, color, etc.
- Compare 2 shapes or objects; tell how they are alike and different
- Explore and build with models of geometry solids
- Recognize basic shapes in a variety of orientations
- Identify, label, and manipulate basic geometric solids and shapes: square, circle, rectangle, triangle, oval, diamond
- Explore symmetry through folding
- Separate into equal sets
- Identify the number in each set/how many left over
- Draw and describe a circle, square, rectangle, triangle
- Identify and describe objects in own environment that depict geometric figures
- Match three dimensional objects that have the same shape
- Describe, estimate and measure using non-standard units
- Explore and solve simple spatial problems using manipulatives and drawings
- Identify and compare 2 and 3 dimensional figures
- Identify and show halves, thirds and fourths

**Benchmark: 1.1.4 Probability & Statistics (Data Analysis)**

**Indicators:**

- Collect and display data with concrete materials
- Generate own ways of recording data
- Explore idea of sampling

- Explore concept of likely and unlikely
- Make and interpret bar and picture graphs based on data collected in own world
- Recognize simple horizontal and vertical bar graphs
- Make and use graphs using real materials, pictures and symbols
- Recognize data on a pie chart
- Recognize ordered pairs on a grid

**Benchmark: 1.1.5 Patterns, Functions & Relationships (Algebra)**

**Indicators:**

- Recognize and create a variety of repeating and increasing patterns (AB, AAB, ABC, AABB)
- Recognize patterns in the environment
- Record patterns
- Interpret patterns

# Mathematical Problem Solving

## STANDARD 2

The student uses mathematics to define and solve problems.

## GRADE 1

**To meet this standard, the student will:**

**Benchmarks: 1.2.1 Investigate Situations**

**Indicators:**

- Conduct explorations
- Search for and identify patterns
- Create and expand repeating and increasing patterns
- Use a variety of strategies and approaches
- Recognize when information is missing or extraneous
- Recognize when an attempted approach is unproductive and tries a new approach

**Benchmark: 1.2.2 Formulate Questions**

**Indicators:**

- Identify questions to be answered
- Define problems in familiar situations
- Identify the unknowns in familiar situations

**Benchmark: 1.2.3 Construct Solutions**

**Indicators:**

- Organize relevant information
- Select and uses tools
- Use a variety of strategies
- Apply appropriate concepts and procedures

# Mathematical Reasoning

**STANDARD 3**

**The student uses mathematical reasoning.**

**GRADE 1**

**To meet this standard, the student will:**

**Benchmarks: 1.3.1 Analyzes Situations**

**Indicators:**

- Interpret information in relatively familiar situations
- Compare and contrast information
- Validate thinking using models, known facts, patterns and relationships

**Benchmark: 1.3.2 Predicts Results:**

**Indicators:**

- Make predictions based on prior experience and known facts

**Benchmark: 1.3.3 Draws Conclusions and Verifies Results**

**Indicators:**

- Test predictions by finding example to support or contradict them
- Supports arguments and justifies results based on experiences
- Check for reasonableness of results using concepts and procedures
- Reflect on and evaluate procedures and results in familiar situations

# Mathematical Communication

**STANDARD 4**

**The student communicates knowledge and understanding in both everyday and mathematical language**

## GRADE 1

To meet this standard, the student will:

### Benchmark: 1.4.1 Gathers Information

#### Indicators:

- Create a plan for collecting information
- Use reading, listening and observation skills to access and extract mathematical information from a variety of sources such as pictures, diagrams, physical models, classmates, oral narratives, and symbolic representations
- Use technology to browse and retrieve mathematical information from a variety of sources

### Benchmark: 1.4.2 Interprets Information

#### Indicators:

- Organize mathematical information
- Clarify mathematical understandings in at least one way – reflecting, verbalizing or discussing
- Share, explain and defend mathematical ideas – concepts and procedures – using terms, language, charts, and graphs that can be clearly understood by a variety of audiences
- Compare data using terms such as more/less, larger/smaller, heavier/lighter

### Benchmark: 1.4.3 Represents and Shares Information

#### Indicators:

- Express ideas using mathematical language and notation such as physical or pictorial models, tables, charts and graphs
- Express mathematical ideas to teachers and peers in appropriate ways

# Mathematical Connections

## STANDARD 5

The student understands how mathematical ideas connect to other subject areas and real-life situations

## GRADE 1

To meet this standard, the student will:

### Benchmarks: 1.5.1 Relates within Mathematics

#### Indicators:

- Recognize mathematical patterns and ideas in familiar situations in other disciplines
- Use mathematical thinking and modeling in familiar situations in other disciplines

**Benchmark: 1.5.2 Relates in Real-Life Situations**

**Indicators:**

- Identify examples of uses of mathematical in own daily life
- Identify examples of uses of mathematics in situations that relate to society

# Concepts and Procedures of Mathematics

## STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

## GRADE 2

**To meet this standard, the student will:**

**Benchmark: 2.1.1 Number Sense & Computation**

**Indicators:**

- Begin to develop the concept of place value by grouping a variety of materials into tens and counting groups of 10 plus leftovers; has multiple experiences counting tens
- Explain strategies used to solve problems mentally
- Add 10 to, or subtract 10 from, any number under 100
- Is fluent in using all number combinations to 20 with missing addends
- Develop efficient strategies for counting large numbers (i.e. counting by 2's, 3's, 5's, and 10's - forward and backwards to 1000)
- Demonstrate conservation of numbers between 30 and 50
- Begin to make estimates for less than 100 objects
- Demonstrate a more sophisticated concept of more and less (compares numbers to tell how many more or less) <or>
- Write and interpret symbolic (+ - =) notation
- Record equations and balanced equations made with manipulatives
- Develop strategies for computing 2 digit numbers with regrouping (addition, subtraction)
- Explore working with a calculator
- Begin to develop understanding of multiplication and division in simple story problems and with manipulatives
- Explore meaning of fractions  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{1}$  and sets of objects in context
- Classify odd or even
- Use physical models, read, write and compare whole numbers and identify appropriate place value to 10,000, including decimals to the hundredths place
- Order positions first through tenth

- Round numbers to the nearest tens and hundreds place

**Benchmark: 2.1.2 Measurement**

**Indicators:**

- Measure using non-standard units (US. and Metric)
- Compare mass on a balance scale
- Read hour, half hour time, quarter-hour and five minute on standard (analog) clock including elapsed time
- Locate specific days on the calendar (i.e., third Tuesday, one week later)
- Estimate measurements
- Recognize, name and record the value of coins and currency through \$1.00
- Select appropriate tool for basic measuring events (time, length, money, weight, temperature, volume, perimeter, and the area)
- Measure line segments using a ruler to the nearest inch and centimeter

**Benchmark: 2.1.3 Geometry & Spatial Sense**

**Indicators:**

- Sorts objects by attributes using a single attribute: number of sides, corners, faces, straight, or curved lines; color, etc.
- Analyzes attributes of shapes; triangle, rectangle, cylinder, cones, pyramid, rectangular prism, cube, sphere, circle and square
- Can create a shape with bilateral symmetry and can use lines of symmetry, translations, rotations and reflections.
- Matches congruent shapes using a variety of materials
- Make, copy and extend patterns with actions, objects, words and numbers

**Benchmark: 2.1.4 Probability & Statistics (Data Analysis)**

**Indicator:**

- Collect, organize, display data with concrete and symbolic representations
- Make simple statements and reasonable predictions from data
- Identify likely and unlikely events
- Compare data using terms such as more/less, larger/smaller, heavier/lighter, and ordinal position words, first, next and last
- Organize and record information using tables, charts and tally marks
- Make a variety of simple graphs
- Describe and analyze a variety of graphs
- Make predictions from experiments using spinners and colored tiles/cubes

**Benchmark: 2.1.5 Patterns, Functions & Relationships (Algebra)**

**Indicators:**

- Recognize, copy, and continue increasing and decreasing patterns (i.e., ababbabbbabbbb)
- Explore predicting “nth” step in building increasing patterns
- Make reasonable estimates about: how many, more, less, nearer to and checks answers

- Given pairs of numbers or tables, describe the functional relationship (e.g. guess my rule)

**Benchmark: 2.1.6 Operations**

**Indicators:**

- Use strategies to add and subtract (vertically and horizontally) numbers up to 20
- Use strategies to add and subtract (vertically and horizontally) 2 and 3 digit numbers with regrouping
- Experience the concept of simple multiplication and its relationship to addition
- Recognize and describe the related facts representing and describing inverse relationships (e.g.,  $3+ \_ = 7$ ,  $7-3= \_$ ,  $7- \_ = 3$ )
- Experience the introduction of simple strategies to divide using manipulatives and its relationship to subtraction
- Add, write and compare the value of numbers up to 1000 using symbols (<, >, =)
- Compare money values in different coin combinations
- Know addition/subtraction family facts through 20
- Identify a missing number in a sequence through 1000

# Mathematical Problem Solving

**STANDARD 2**

**The student uses mathematics to define and solve problems.**

**GRADE 2**

**To meet this standard, the student will:**

**Benchmark: 2.2.1 Investigate Situations**

**Indicators:**

- Conduct explorations
- Search for patterns
- Use a variety of strategies and approaches
- Recognize when information is missing or extraneous
- Recognize when an attempted approach is unproductive and tries a new approach

**Benchmark: 2.2.2 Formulates Questions**

**Indicators:**

- Identify questions to be answered
- Define problems in familiar situations
- Identify the unknowns in familiar situations

**Benchmark: 2.2.3 Constructs Solutions**

**Indicators:**

- Organize relevant information
- Select and uses tools
- Use a variety of strategies
- Apply appropriate concepts and procedures

## Mathematical Reasoning

### STANDARD 3

**The student uses mathematical reasoning.**

### GRADE 2

**To meet this standard, the student will:**

**Benchmarks: 2.3.1 Analyzes Situations**

**Indicators:**

- Interpret information in relatively familiar situations
- Compare and contrast information
- Validate thinking using models, known facts, patterns and relationships

**Benchmark: 2.3.2 Predicts Results:**

**Indicators:**

- Make predictions based on prior experience and known facts

**Benchmark: 2.3.3 Draws Conclusions and Verifies Results**

**Indicators:**

- Test predictions by finding example to support or contradict them
- Supports arguments and justifies results based on experiences
- Check for reasonableness of results using concepts and procedures more than one way
- Reflect on and evaluate procedures and results in familiar situations

## Mathematical Communication

### STANDARD 4

**The student communicates knowledge and understanding in both everyday and mathematical language**

## GRADE 2

To meet this standard, the student will:

### Benchmark: 2.4.1 Gathers Information

#### Indicators:

- Create a plan for collecting information
- Use reading, listening and observation skills to access and extract mathematical information from a variety of sources such as pictures, diagrams, physical models, classmates, oral narratives, and symbolic representations
- Use technology to browse, organize and retrieve mathematical information from a variety of sources

### Benchmark: 2.4.2 Interprets Information

#### Indicators:

- Compare data using terms such as more/less, larger/smaller, and heavier/lighter
- Organize and interpret mathematical information
- Share, explain and defend mathematical ideas – concepts and procedures – using terms, language, charts, and graphs that can be clearly understood by a variety of audiences
- Sort, compare, interpret, and record data to create simple charts and graphs
- Analyze or extends pattern or develops general rule from data.

### Benchmark: 2.4.3 Represents and Shares Information

#### Indicators:

- Explain, write, share and justify answers and procedures
- Express mathematical ideas using terms, language, equations, organized lists, labeled diagrams, tables and graphs

# Mathematical Connections

## STANDARD 5

The student understands how mathematical ideas connect to other subject areas and real-life situations

## GRADE 2

To meet this standard, the student will:

### Benchmarks: 2.5.1 Relates within Mathematics

#### Indicators:

- Recognize mathematical patterns and ideas in familiar situations in other disciplines

- Use mathematical thinking and modeling in familiar situations in other disciplines
- Extends mathematical concept to a more difficult situation
- Identifies general rule and uses it in two or more examples beyond the problem

**Benchmark: 2.5.2 Relates in Real-Life Situations**

**Indicators:**

- Identify examples of uses of mathematical in own daily life
- Identify examples of uses of mathematics in situations that relate to society

# Concepts and Procedures of Mathematics

## STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

## GRADE 3

**To meet this standard, the student will:**

**Benchmark: 3.1.1 Understand and apply the concepts and procedures from number sense and computation**

**Indicators:**

- Identify concrete and pictorial representations of whole and fractional numbers
- Recognize place value in representations of whole numbers of increasing size
- Understand properties of the whole number system at the concrete level
- Understand and apply the concepts of comparing, sequencing, and ordering
- Show understanding of whole number operations of addition, subtraction, multiplication, and division at the concrete level
- Add and subtract whole numbers with and without grouping
- Use mental mathematics, pencil and paper, or calculator as appropriate to the task
- Understand that there are situations in which estimation is helpful
- Use estimation to predict computation results
- Use estimation to determine reasonableness of answers

**Benchmark: 3.1.2 Measurement**

**Indicators:**

- Use measurable attributes such as length, perimeter, area, volume, weight, money, time and temperature to describe and make comparisons
- Understand the concepts of perimeter and area
- Explore estimation of measurement and compare to actual measurement

- Know how to use estimation to predict and determine when measurements are reasonable
- Understand use appropriate units of measure for money, time, length, area, volume, mass, and temperature
- Use appropriate tool for basic measuring events: time, length, area, volume, mass, and temperature
- Understand benefits of using standard units to measure length, width, etc.

**Benchmark: 3.1.3 Geometry and Spatial Sense**

**Indicators:**

- Explore properties of one, two, and three dimensional objects
- Identify, label, and manipulate three dimensional shapes (cone, sphere, cube)
- Understand apply concepts of symmetry and similarity, and congruence
- Construct simple shapes using appropriate tools

**Benchmark: 3.1.4 Probability and Statistics (Data Analysis)**

**Indicators:**

- Collects, organizes, displays, and makes statements about data
- Describes information presented in bar, line and pictorial graphs
- Investigates and explains likely, unlikely, and equally likely

**Benchmark: 3.1.5 Patterns, Functions and Relationships (algebraic sense)**

**Indicators:**

- Understand the use of guess and check in the search for patterns
- Use standard notation in reading and writing open sentences
- Describe equalities and inequalities using appropriate symbols( $>$ , $<$ , $=$ )
- Recognize, create and expand patterns
- Know how to solve simple equations at the concrete level
- Evaluate simple equations at the concrete level

# Mathematical Problem Solving

**STANDARD 2**

**The student uses mathematics to define and solve problems.**

**GRADE 3**

**To meet this standard, the student will:**

**Benchmark: 3.2.1 Investigate situations**

**Indicators:**

- Conduct explorations
- Search for patterns
- Use a variety of strategies and approaches
- Recognize when information is missing or extraneous
- Recognize when an attempted approach is unproductive and tries a new approach

**Benchmark: 3.2.2 Formulate questions**

**Indicators:**

- Identify questions to be answered
- Define problems in familiar situations
- Identify the unknowns in familiar situations

**Benchmark: 3.2.3 Construct Solutions**

**Indicators:**

- Organize relevant information
- Select and uses tools
- Use a variety of strategies
- Apply appropriate concepts and procedures

## Mathematical Reasoning

**STANDARD 3**  
**The student uses mathematical reasoning.**

**GRADE 3**

**To meet this standard, the student will:**

**Benchmark: 3.3.1 Analyze situations**

**Indicators:**

- Interpret information in relatively familiar situations
- Compare and contrast information
- Validate thinking using models, know facts, patterns and relationships

**Benchmark: 3.3.2 Predict results**

**Indicators:**

- Make predictions based on prior experience and known facts

**Benchmark: 3.3.3 Draw conclusions and verify results**

**Indicators:**

- Check for reasonableness of results
- Guess and check
- Use opposite operation to check work
- Recognize extra data as unnecessary
- Support arguments and justify results

# Mathematical Communication

**STANDARD 4**

**The student communicates knowledge and understanding in both everyday and mathematical language**

## GRADE 3

**To meet this standard, the student will:**

**Benchmark: 3.4.1 Gather information**

**Indicators:**

- Create a plan for collecting information
- Use reading, listening and observation skills to access and extract mathematical information from a variety of sources such as pictures, diagrams, physical models, classmates, oral narratives, and symbolic representations
- Use technology to browse and retrieve mathematical information from a variety of sources

**Benchmark: 3.4.2 Interpret information**

**Indicators:**

- Organize mathematical information
- Clarify mathematical understandings in at least one way- reflecting, verbalizing or discussing
- Explain and defend mathematical ideas- concepts and procedures- using terms, language, charts, and graphs that can be clearly understood by a variety of audiences

**Benchmark: 3.4.3 Represent and share information**

**Indicators:**

- Express ideas using mathematical language and notation such as physical or pictorial models, tables, charts and graphs or open sentences
- Express mathematical ideas to teachers and peers in appropriate ways

## Mathematical Connections

### STANDARD 5

**1The student understands how mathematical ideas connect to other subject areas and real-life situations**

### GRADE 3

**To meet this standard, the student will:**

**Benchmark: 3.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Recognize mathematical patterns and ideas in familiar situations in other disciplines
- Use mathematical thinking and modeling in familiar situations in other disciplines
- Describe examples of contributions of different cultures to the historical development of number systems and patterns

**Benchmark: 3.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Make connections between math concepts and other subjects
- Recognize mathematical patterns and ideas in familiar situations in other disciplines
- Use mathematical thinking and modeling in familiar situations in other disciplines

**Benchmark: 3.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Identify examples of uses of mathematics in own daily life
- Identify examples of uses of mathematics in situations that relate to society

# Concepts and Procedures of Mathematics

**STANDARD 1**  
**The student understands and applies the concepts and procedures of mathematics.**

## GRADE 4

**To meet this standard, the student will:**

**Benchmark: 4.1.1 Understand and apply concepts and procedures from number sense and computation**

### **Indicators:**

- Use objects, pictures, or symbols to demonstrate understanding of whole and fractional numbers, place value in whole numbers, and properties of the whole number system
- Identify, compare, and order whole numbers and simple fractions
- Understand whole number operations of addition, subtraction, multiplication, and division at the concrete level
- Know how to add, subtract, multiply and divide whole numbers
- Understand how to estimate, approximate, round off, and/or use exact precision as appropriate and necessary in a calculation
- Understand how addition, subtraction, multiplication, and division relate to one another
- Understand division as sharing or grouping
- Interpret the remainder appropriately
- Understand that there are situations in which estimation is helpful
- Use estimation to predict computation results
- Use estimation to determine reasonableness of answers

**Benchmark: 4.1.2 Measurement**

### **Indicators:**

- Use measurable attributes such as length, perimeter, area, volume, weight, money, time and temperature to describe and compare objects
- Estimate to predict and determine when measurements are reasonable (estimating the length of the playground by pacing it off)
- Understand the benefits of using standard units of measurement for measuring length, area, and volume

- Understand appropriate units of measure for time, money, length, area, volume, mass, and temperature
- Use appropriate tools for measuring time, money, length, area, volume, mass, and temperature

**Benchmark: 4.1.3 Geometry and Spatial Sense**

**Indicators:**

- Identify one, two, and three dimensions
- Recognize geometric shapes in the surrounding environment, for example identify within windows
- Describe the location of objects relative to each other on grids or maps
- Understand concepts of parallel and perpendicular
- Understand and construct simple geometric transformations using slides, flips, or turns
- Understand concept of symmetry, congruence and similarity

**Benchmark: 4.1.4 Probability and Statistics (Data Analysis)**

**Indicators:**

- Understand the difference between certain and uncertain events
- Know that data can be represented in different forms such as tabulations of events, objects, or occurrences
- Collect data in an organized way
- Organize and display data in numerical and graphical forms such as tables, charts, pictographs, and bar graphs
- Predict outcomes of simple activities and compare predictions to experimental results
- Understand and make inferences based on experimental results using coins, number cubes, spinners, etc.

**Benchmark: 4.1.5 Patterns, Functions and Relationships (algebraic sense)**

**Indicators:**

- Understand how to extend patterns by identifying a rule that generates the pattern and use variety of strategies to identify patterns
- Know how to solve simple equations at the concrete level
- Evaluate simple equations

# Mathematical Problem Solving

**STANDARD 2**

**The student uses mathematics to define and solve problems.**

## GRADE 4

To meet this standard, the student will:

**Benchmark: 4.2.1 Investigate situations**

**Indicators:**

- Conduct explorations
- Search for patterns
- Use a variety of strategies and approaches
- Recognize when information is missing or extraneous
- Recognize when an attempted approach is unproductive and try a new approach
- Restate the question in a mathematical problem

**Benchmark: 4.2.2 Formulate questions**

**Indicators:**

- Identify questions to be answered
- Define problems in familiar situations
- Identify the unknowns in familiar situations

**Benchmark: 4.2.3 Construct Solutions**

**Indicators:**

- Organize relevant information
- Select and use appropriate mathematical tools
- Apply appropriate methods, operations and processes to construct a solution

# Mathematical Reasoning

## STANDARD 3

The student uses mathematical reasoning.

## GRADE 4

To meet this standard, the student will:

**Benchmark: 4.3.1 Analyze situations**

**Indicators:**

- Predict reasonable answers
- Gather, and interpret, and compare information from a variety of sources

**Benchmark: 4.3.2 Predict results**

**Indicators:**

- Make predictions based on prior experience and known facts

**Benchmark: 4.3.3 Draw conclusions and verify results**

**Indicators:**

- Check for reasonableness of results and include examples to support or contradict
- Guess and check
- Use opposite operation to check work
- Recognize extra data as unnecessary
- Support arguments and justify results using inductive reasoning

## Mathematical Communication

### STANDARD 4

**The student communicates knowledge and understanding in both everyday and mathematical language.**

### GRADE 4

**To meet this standard, the student will:**

**Benchmark: 4.4.1 Gather information**

**Indicators:**

- Create a plan for collecting information
- Use reading, listening and observation skills to access and extract mathematical information from a variety of sources diagrams, pictures, etc.
- Use technology to select and retrieve information from a variety of sources

**Benchmark: 4.4.2 Interpret information**

**Indicators:**

- Organize mathematical information
- Clarify mathematical understandings by reflecting, verbalizing, and discussing

**Benchmark: 4.4.3 Represent and share information**

**Indicators:**

- Express verbally, explain, write, share and justify answers and procedures
- Sort, compare, interpret, and record data to create charts and graphs

## Mathematical Connections

### STANDARD 5

**The student understands how mathematical ideas connect to other subject areas and real-life situations**

### GRADE 4

**To meet this standard, the student will:**

**Benchmark: 4.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Link math concepts and procedures in different mathematical content areas
- See relationship between selected part of mathematics
- Recognize equivalent mathematical models and representations in familiar situations

**Benchmark: 4.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Make connections between math concepts and other subjects
- Recognize mathematical patterns and ideas in familiar situations in other disciplines
- Use mathematical thinking and modeling in familiar situations in other disciplines
- Describe examples of contributions to the development of math such as the contributions of women, men and different cultures

**Benchmark: 4.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Identify examples of uses of mathematics in own daily life
- Identify examples of uses of mathematics in situations that relate to society and career settings

# Concepts and Procedures of Mathematics

## STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

## GRADE 5

To meet this standard, the student will:

**Benchmark: 5.1.1 Number Sense and Computation**

### Indicators:

- Use objects, pictures, or symbols to demonstrate understanding of fractions, decimals, percents, and place value in decimals
- Compare and order 5 digit whole numbers, fractions (denominators of 2, 3, 4, 8, 12) and decimals (to hundredths)
- Understand operations of whole numbers
- Add, subtract, multiply, and divide fractions and decimals using rules for order of operation
- Use estimation to predict computation results and to determine the reasonableness of answers involving rational numbers, for example, estimating a tip
- Use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving whole numbers fractions and decimals
- Identify situations where estimation is useful and sufficient using a variety of strategies to estimate
- Understand the concepts of prime and composite numbers, factors and multiples, and divisibility rules (2, 3, 5, 9, 10)

**Benchmark: 5.1.2 Measurement**

### Indicators:

- Understand the relationship among perimeter, area, and volume
- Measure objects and distances directly or using indirect methods.

- Use estimation to obtain reasonable approximations, for example, estimating the length and width of the playground to approximate its area
- Understand that measurement precision is related to unit used
- Understand the relationship among units within both the customary and metric systems
- Understand the benefits of standard units of measurement for both direct and indirect measurement

**Benchmark: 5.1.3 Geometry and Spatial Sense**

**Indicators:**

- Identify geometric shapes and relationships
- Identify and describe objects in the surrounding environment in geometric terms, for example, describe the triangles that make up a bridge structure
- Describe the location of objects on coordinate grids
- Understand and identify properties and relationships of plane geometry including ray; angle; isosceles; equilateral; and degrees in a circle, triangle, or quadrilateral
- Identify and construct symmetric, congruent, and similar figures
- Understand and construct simple geometric transformations using combinations of slides, flips, or turns
- Use a compass and straightedge, and/or computer software to perform geometric constructions

**Benchmark: 5.1.4 Probability and Statistics (Data Analysis)**

**Indicators:**

- Understand the difference between certain and uncertain events
- Understand the procedures for counting outcomes to determine probabilities
- Understand and use experiments to investigate uncertain events
- Organize and display data in appropriate forms such as frequency tables or circles graphs
- Collect data in an organized way
- Calculate and use mean, median, and mode as appropriate in describing a set of data

**Benchmark: 5.1.5 Patterns, Functions and Relationships (algebraic sense)**

**Indicators:**

- Recognize, create, and extend patterns and sequences
- Represent number patterns with tables, graphs, and rules
- Represent equalities and inequalities symbolically using = <, >
- Set up and solve single-variable equations ( $6 + D = 9$ )

# Mathematical Problem Solving

**STANDARD 2**

**The student uses mathematics to define and solve problems.**

## GRADE 5

**To meet this standard, the student will:**

**Benchmark: 5.2.1 Investigate situations**

**Indicators:**

- Conduct explorations
- Search systematically for patterns in simple situations
- Develop and use a variety of strategies and approaches
- Identify missing or extraneous information
- Recognize the need to modify or abandon an unproductive approach

**Benchmark: 5.2.2 Formulate questions**

**Indicators:**

- Identify questions to be answered
- Define problems in familiar situations
- Identify the unknowns in familiar situations

**Benchmark: 5.2.3 Construct Solutions**

**Indicators:**

- Organize relevant information from relevant sources
- Select and use appropriate mathematical tools
- Applies appropriate concepts and procedures

# Mathematical Reasoning

## STANDARD 3

**The student uses mathematical reasoning.**

## GRADE 5

**To meet this standard, the student will:**

**Benchmark: 5.3.1 Analyze situations**

**Indicators:**

- Interpret information in relatively familiar situations
- Compare and contrast information
- Validate thinking using models, know facts, patterns and relationships

**Benchmark: 5.3.2 Predict results**

**Indicators:**

- Make conjectures and inferences based on analysis of relatively familiar problems

**Benchmark: 5.3.3 Draw conclusions and verify results**

**Indicators:**

- Test conjectures and inferences and explain why they are true or false
- Supports arguments and justifies results based on experiences
- Check for reasonableness of results
- Reflects on and evaluates procedures and results in familiar situations

## Mathematical Communication

### STANDARD 4

The student communicates knowledge and understanding in both everyday and mathematical language.

### GRADE 5

**To meet this standard, the student will:**

**Benchmark: 5.4.1 Gather information**

**Indicators:**

- Create a plan for collecting information
- Use reading, listening and observation skills to access and extract mathematical information from a variety of sources such as pictures, diagrams, physical models, classmates, oral narratives, and symbolic representations
- Use technology to browse and retrieve mathematical information from a variety of sources

**Benchmark: 5.4.2 Interpret information**

**Indicators:**

- Organize and clarify mathematical information by reflecting, verbalizing, discussing, or writing

**Benchmark: 5.4.3 Represent and share information**

**Indicators:**

- Express ideas using mathematical language and notation such as physical or pictorial models, tables, charts and graphs or open sentences
- Express mathematical ideas to teachers and peers in appropriate ways

## Mathematical Connections

### STANDARD 5

**The student understands how mathematical ideas connect to other subject areas and real-life situations**

### GRADE 5

**To meet this standard, the student will:**

**Benchmark: 5.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Connect conceptual and procedural understandings among familiar and different mathematical content areas
- Relate and use different mathematical models and representations for the same situation

**Benchmark: 5.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Identify mathematical patterns and ideas in other disciplines
- Use mathematical thinking and modeling in other disciplines
- Describe examples of contributions to the development of mathematics such as the contributions of men, women and different cultures

**Benchmark: 5.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Recognize the extensive use of mathematics outside the classroom, for example, in banking or sports statistics
- Investigate the use of mathematics within several occupational / career areas of interest

## Concepts and Procedures of Mathematics

**STANDARD 1**  
**The student understands and applies the concepts and procedures**  
**of mathematics.**

**GRADE 6**

**To meet this standard, the student will:**

**Benchmark: 6.1.1. Number Sense and Computation**

**Indicators:**

- Demonstrate understanding of fractions with denominators 2, 3, 4, 5, 10 and 12 and 3 place decimals, and place values by using pictures and symbols
- Compare and order 6 digit whole numbers, fractions, denominators up to 12, and decimals
- Understand operations on rational and whole numbers
- Add, subtract, multiply, and divide non-negative fractions with denominators to 8 and 3 place decimals using rules for order of operation
- Use estimation to predict computation results and to determine whether the computed answer is reasonable
- Use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving rational numbers and whole numbers
- Identify situations involving rational numbers in which estimation is sufficient and computation is not required
- Understand and use basic exponents
- Understand the concepts of prime and composite numbers, factors and multiples, and divisibility rules for 2, 3, 4, 5, 6, 9, 10 and prime factorization
- Understand the concepts of ratio, rates, and direct proportion

**Benchmark: 6.1.2 Measurement**

**Indicators:**

- Understand when to use and how to calculate perimeter, area, and volume and the relationship among them. Squares, rectangles, parallelograms, triangles, trapezoids
- Measure objects and events directly or use indirect methods
- Understand the concept of rate (as related to measurement) and how to calculate those rates and determine unit rate
- Use estimation to obtain reasonable approximations, for example, estimating the length and width of the playground to approximate its area
- Select and use units that will provide an appropriate degree of precision, for example, using meters vs. kilometers
- Understand that precision is related to the unit of measurement used
- Understand the relationship among units within both the customary and metric systems
- Understand the benefits of standard units of measurement

**Benchmark: 6.1.3 Geometry and spatial sense**

**Indicators:**

- Use multiple attributes to describe geometric shapes
- Identify and describe objects in the surrounding environment in geometric terms, for example, describe the triangles that make up a bridge structure
- Describe location of objects on coordinate grids
- Understand and identify properties of plane geometry including ray; angle; isosceles; equilateral; and degrees in a circle, triangle, or quadrilateral
- Construct symmetric, congruent, and similar figures
- Understand and construct simple geometric transformations using combinations of slides, flips, or turns
- Use a compass and straightedge, and/or computer software to perform geometric constructions

**Benchmark: 6.1.4 Probability and Statistics (Data Analysis)**

**Indicators:**

- Know how to calculate numerical measures of uncertainty for simple events
- Understand procedures for counting outcomes to determine probabilities
- Know how to conduct experiments and simulations and to compare results with mathematical expectations
- Predict outcomes of experiments and simulations and compare the predictions to experimental results
- Understand and make inferences based on experimental results
- Organize and display data in appropriate forms such as frequency tables, circle graphs, bar graphs and histograms
- Collect a random sample of data that represents a described population
- Calculate and use mean, median, and mode as appropriate in describing a set of data
- Identify how different types of statistics can be used to support different points of view

**Benchmark: 6.1.5 Patterns, Functions and Relationships (algebraic sense)**

**Indicators:**

- Recognize, create, and extend patterns and sequences
- Represent number patterns with tables, graphs and rules
- Represent equalities and inequalities symbolically using  $=$ ,  $\geq$ ,  $>$ ,  $<$ ,  $\leq$ ,
- understand and use variables in simple equations and formulas, for example,  $3x = 18$
- Evaluate simple expressions using manipulatives
- Set up and solve simple one-step equations, for example  $3 + x = s$

# Mathematical Problem Solving

## STANDARD 2

The student uses mathematics to define and solve problems.

### GRADE 6

To meet this standard, the student will:

**Benchmark: 6.2.1 Investigate situations**

**Indicators:**

- Conduct explorations
- Search systematically for patterns in simple situations
- Develop and use a variety of strategies and approaches
- Identify missing or extraneous information
- Recognize the need to modify or abandon an unproductive approach

**Benchmark: 6.2.2 Formulate questions and define the problem**

**Indicators:**

- Identify questions to be answered in new situations
- Define problems in new situations
- Identify the unknowns in new situations

**Benchmark: 6.2.3 Construct solutions**

**Indicators:**

- Organize relevant information from multiple sources
- Select and use appropriate mathematical tools
- Apply appropriate methods, operations, and processes to construct a solution

# Mathematical Reasoning

## STANDARD 3

The student uses mathematical reasoning.

### GRADE 6

To meet this standard, the student will:

**Benchmark: 6.3.1 Analyze situations**

**Indicators:**

- Interpret, compare, and contrast information from a variety of sources
- Validate thinking and mathematical ideas using models, known facts, patterns, and relationships

**Benchmark: 6.3.2 Predict results and make inferences**

**Indicators:**

- Make conjectures and inferences based on analysis of familiar and new problem situations

**Benchmark: 6.3.3 Draw conclusions and verify results**

**Indicators:**

- Support arguments and justify results using logical reasoning
- Test conjectures and inferences and explain why they are accurate or not
- Check for reasonableness of results
- Reflect and evaluate on procedures and results in new problem situations

# Mathematical Communication

**STANDARD 4**

**The student communicates knowledge and understanding in both everyday and mathematical language.**

**GRADE 6**

**To meet this standard, the student will:**

**Benchmark: 6.4.1 Gather information**

**Indicators:**

- Develop a plan for collecting information
- Use reading, listening, and observation skills to access and extract mathematical information from multiple sources such as pictures, diagrams, physical models, oral narratives, and symbolic representations
- Choose appropriate **available** technology to browse, select, and retrieve relevant mathematical information from a variety of sources

**Benchmark: 6.4.2 Organize and interpret information**

**Indicators:**

- Organize and clarify mathematical information by reflecting, verbalizing, discussing, or writing

**Benchmark: 6.4.3 Represent and share information**

**Indicators:**

- Clearly and effectively express or present ideas and situations using both everyday and mathematical language such as models, tables, charts, graphs, written reflection, or algebraic notation
- Express mathematical ideas to teachers, peers and parents in appropriate ways

## Mathematical Connections

**STANDARD 5**

**The student understands how mathematical ideas connect to other subject areas and real-life situations**

**GRADE 6**

**To meet this standard, the student will:**

**Benchmark: 6.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Connect conceptual and procedural understandings among familiar and different mathematical content areas
- Relate and use different mathematical models and representations for the same situation

**Benchmark: 6.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Identify mathematical patterns and ideas in other disciplines
- Use mathematical thinking and modeling in other disciplines
- Describe examples of contributions to the development of mathematics such as the contributions of women, men, and different cultures

**Benchmark: 6.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Recognize the extensive use of mathematics outside the classroom, for example, in banking or sports statistics
- Investigate the use of mathematics within several occupational/career areas of interest

# Concepts and Procedures of Mathematics

## STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

## GRADE 7

To meet this standard, the student will:

**Benchmark: 7.1.1      Number Sense and Computation**

### Indicators:

- Use pictures, diagrams and symbols to demonstrate understanding of fractions with denominators up to 20, 4 place decimals, percents, place value in non-negative decimals, zero and properties of the rational number system
- Compare and order 7 digit whole numbers, fractions with denominators to 20 and 4 place decimals
- Demonstrate understanding of adding, subtracting, multiplying and dividing positive whole number, fractions, with denominators to 20, 4 place decimals and '0'
- Understand and use order of operations on whole numbers, fractions and decimals
- Use estimation to predict computation results with 4 operations on whole numbers and to determine the reasonableness of answers involving rational numbers, for example, estimating a tip
- Use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving rational numbers and whole numbers
- Identify situations involving rational numbers in which estimation is sufficient and computation is not required
- Understand the concepts of prime and composite numbers, factors and multiples, divisibility rules for 2, 3, 4, 5, 6, 9 and 10, and prime factorization
- Understand and use exponents
- Understand the concepts of ratio, rate, direct proportion and solve proportion problems
- Convert numbers into decimals, fractions or percents and interchange among the three
- Solve problems involving percents (percent increase and decrease)
- Understand the concepts and operations on integers

**Benchmark: 7.1.2      Measurement**

### Indicators:

- Understand concept and relationship of perimeter, area and volume of rectangles, squares, parallelograms, trapezoids, triangles and circles and when to apply each to a given situation
- Measure objects directly or using indirect methods such as formulas
- Use estimation to obtain reasonable approximations for measurement

- Understand that precision is related to the unit of measurement used and the calibration of the measurement tool used
- Select and use units that will provide an appropriate degree of precision
- Understand the relationship among units within both the customary and metric systems
- Understand the use and benefits of standard units of measurement for both direct and indirect measurement
- Understand the concept of rate, ratio and how to calculate rates and determine unit rates

**Benchmark: 7.1.3 Geometry and Spatial Sense**

**Indicators:**

- Use multiple attributes to describe geometric shapes
- Identify and describe objects in the surrounding environment in geometric terms
- Describe location of objects on coordinate grids
- Understand and identify properties and relationships of plane geometry including ray; angle; isosceles; equilateral; and degrees in a circle, triangle, or quadrilateral
- Construct symmetric, congruent and similar figures
- Understand and construct simple geometric transformations using combinations of slides, flips, or turns
- Use a compass and straightedge, and/or computer software to perform geometric constructions
- Understand how to find perimeter and area of rectangle, triangles and circle

**Benchmark: 7.1.4 Probability and Statistics (Data Analysis)**

**Indicators:**

- Know how to calculate numerical measures of uncertainty for simple events
- Understand procedures for counting outcomes to determine probabilities
- Know how to conduct experiments and simulations and to compare results with mathematical expectations
- Predict outcomes of experiments and simulations and compare the predictions to experimental results
- Understand and make inferences based on experimental results
- Organize and display data in appropriate forms
- Collect a random sample of data that represents a described population
- Calculate and use mean, median, and mode as appropriate in describing a set of data
- Identify how different types of statistics can be used to support different points of view

**Benchmark: 7.1.5 Patterns, Functions and Relationships (algebraic sense)**

**Indicators:**

- Recognize, create, and extend patterns and sequences
- Represent number patterns with tables, graphs, and rules
- Represent equalities and inequalities symbolically using  $=$ ,  $>$ ,  $<$ ,  $\leq$ ,  $\geq$
- Understand and use variables in simple equations
- Evaluate simple expressions

- Set up and solve one-step equations

# Mathematical Problem Solving

## STANDARD 2

The student uses mathematics to define and solve problems.

## GRADE 7

To meet this standard, the student will:

**Benchmark: 7.2.1 Investigate situations**

**Indicators:**

- Conduct explorations
- Search systematically for patterns in simple situations
- Develop and use a variety of strategies and approaches
- Identify missing or extraneous information
- Recognize the need to modify or abandon an unproductive approach

**Benchmark: 7.2.2 Formulate questions and define the problem**

**Indicators:**

- Identify questions to be answered in new situations
- Define problems in new situations
- Identify the unknowns in new situations

**Benchmark: 7.2.3 Construct solutions**

**Indicators:**

- Organize relevant information from multiple sources
- Select and use appropriate mathematical tools
- Apply appropriate methods, operations, and processes to construct a solution

# Mathematical Reasoning

## STANDARD 3

The student uses mathematical reasoning.

## GRADE 7

To meet this standard, the student will:

**Benchmark: 7.3.1 Analyze situations**

**Indicators:**

- Interpret, compare, and contrast information from a variety of sources
- Validate thinking and mathematical ideas using models, known facts, patterns, relationships, and counter-examples

**Benchmark: 7.3.2 Predict results and make inferences**

**Indicators:**

- Predict results and make inferences based on new problem situations

**Benchmark: 7.3.3 Draw conclusions and verify results**

**Indicators:**

- Test conjectures and inferences and explain why they are accurate or not
- Verify results and explain why they are accurate or not
- Support arguments and justify results using logical reasoning
- Check for reasonableness of results
- Reflect and evaluate on procedures and results in new problem situations

# Mathematical Communication

## STANDARD 4

The student communicates knowledge and understanding in both everyday and mathematical language.

## GRADE 7

To meet this standard, the student will:

**Benchmark: 7.4.1 Gather information**

**Indicators:**

- Develop a plan for collecting information
- Use reading, listening, and observation skills to access and extract mathematical information from multiple sources such as pictures, diagrams, physical models, oral narratives, and symbolic representations
- Choose appropriate **available** technology to browse, select, and retrieve relevant mathematical information from a variety of sources

**Benchmark: 7.4.2 Organize and interpret information**

**Indicators:**

- Organize and clarify mathematical information by reflecting, verbalizing, discussing, or writing

**Benchmark: 7.4.3 Represent and share information**

**Indicators:**

- Clearly and effectively express or present ideas and situations using both everyday and mathematical language, such as models, tables, charts, graphs, written reflection, or algebraic notation
- Express mathematical ideas to teachers, parents, peers and younger students in appropriate ways

## Mathematical Connections

### STANDARD 5

**The student understands how mathematical ideas connect to other subject areas and real-life situations**

### GRADE 7

**To meet this standard, the student will:**

**Benchmark: 7.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Connect conceptual and procedural understandings among different mathematical content areas
- Relate and use different mathematical models and representations for the same situation

**Benchmark: 7.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Identify mathematical patterns and ideas in other disciplines
- Use mathematical thinking and modeling in other disciplines
- Describe examples of contributions to the development of mathematics such as the contributions of women, men, and different cultures

**Benchmark: 7.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Recognize the extensive use of mathematics outside the classroom, for example, in banking or sports statistics
- Investigate the use of mathematics within several occupational/career areas of interest

# Concepts and Procedures of Mathematics

## STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

## PRE-ALGEBRA GRADES 7 OR 8

To meet this standard, the student will:

**Benchmark: 7/8.1.1 Number Sense and Computation**

**Indicators:**

- Understand and use operations on all whole numbers, fractions and decimals
- Compare and order 7 digit whole numbers, fractions with any denominator, and 5 place decimals
- Understand and use order of operations on real numbers
- Use estimation to predict computation results and to determine the reasonableness of answers involving real numbers
- Use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving real numbers
- Identify situations involving real numbers in which estimation is sufficient and computation is not required
- Understand the concepts of prime and composite numbers, factors and multiples, and divisibility rules for 2, 3, 4, 5, 6, 9, and 10
- Compute with exponents on real numbers
- Understand operations on expressions with exponents
- Understand the concepts of ratio, rate, direct proportion and solve proportion problems
- Convert numbers into decimals, fractions or percents and interchange among the three
- Solve problems involving percents (percent increase and decrease)
- Understand how to simplify expressions with brackets and parentheses (order of operations on real numbers)
- Understand the concepts of inequalities and how to symbolically represent them on a number line
- Understand how to add, subtract, multiply, and divide using substituted values
- Understand how to evaluate and simplify number and variable expressions

**Benchmark: 7/8.1.2 Measurement**

**Indicators:**

- Understand the concepts and relationship of perimeter, and area, and volume of rectangles, square, parallelograms, trapezoids, triangles, circles and prisms and when to apply to a given situation
- Be able to measure objects directly or by using indirect methods (such as formulas)
- Use estimation to obtain reasonable approximations for measurements
- Understand that precision is related to the unit of measurement and the tool used
- Select and use units that will provide an appropriate degree of precision (e.g. meters vs. kilometers)
- Understand the relationships among units within both customary and metric systems
- Understand the standard units of measurements for both direct and indirect measurements
- Understand the concept of rate, ratio and how to calculate unit ratios

**Benchmark: 7/8.1.3 Geometry and Spatial Sense**

**Indicators:**

- Use multiple attributes to describe geometric shapes
- Identify and describe objects in the surrounding environment in geometric terms
- Understand how to graph ordered pairs of numbers in a coordinate plane
- Understand how to find the coordinates and quadrants of any point in the plane
- Know how to graph any linear equation in two variables
- Know how to find the intercepts of a linear function
- Understand and construct symmetrical, congruent and similar figures
- Construct multiple geometric transformations using combinations of translation, reflection, or rotation
- Understand how to find perimeter and area of rectangles, triangles, circles, trapezoids, parallelogram and complex figures
- Understand how to find volume and surface area of three dimensional shapes

**Benchmark: 7/8.1.4 Probability and Statistics (Data Analysis)**

**Indicators:**

- Know how to calculate probability for simple events
- Understand and use appropriate counting procedures to determine probabilities
- Use both experimental and theoretical methods to determine probabilities
- Design and conduct experiments to verify or disprove predictions
- Predict outcomes of experiments and simulations and compare the prediction to experimental results
- Understands and makes inferences based on the analysis of experimental results
- Organize and display data in appropriate forms
- Collect data using appropriate methods and technology

- Calculate and use mean, median, mode and range as appropriate in describing a set of data
- Calculate and use the different measures and central tendency, variability, and range as appropriate in describing sets of data
- Identify different types of statistics can be used to support different points of view
- Understand the properties of dependent and independent events

**Benchmark: 7/8.1.5 Patterns, Functions and Relationships (algebraic sense)**

**Indicators:**

- Recognize, create, and extend patterns and sequences
- Understand how to add/subtract/multiply/divide variables to simplify an expression
- Represent number patterns and tables, graphs, and rules
- Represent inequalities and equalities algebraically using  $=, >, <, \geq, \leq$
- Use variables in simple equations, inequalities and formulas
- Understand how to use the distributive property of multiplication over addition/subtraction with variables
- Know how to find solution sets of open sentences in two variables
- Know how to add, subtract, multiply and divide algebraic monomials
- Apply commutative and associative properties
- Evaluate and simplify expressions
- Create and solve one, two and three-step equations and inequalities

# Mathematical Problem Solving

**STANDARD 2**

**The student uses mathematics to define and solve problems.**

**PRE-ALGEBRA GRADES 7 OR 8**

**To meet this standard, the student will:**

**Benchmark: 7/8.2.1 Investigate situations**

**Indicators:**

- Conduct explorations
- Search for patterns systematically
- Analyze and use multiple strategies
- Identify information within the problem which is useful for reaching the solution
- Analyze an unproductive approach and attempt to modify it or try a new approach

**Benchmark: 7/8.2.2 Formulate questions and define the problem**

**Indicators:**

- Define problems/unknowns in new situations
- Identify the unknowns and questions to be answered in new situations

**Benchmark: 7/8.2.3 Construct solutions**

**Indicators:**

- Organize and synthesize information from multiple sources
- Select and use appropriate mathematical operations and tools
- Apply appropriate methods, operations, and processes to construct a solution
- Make a diagram to solve problems

## Mathematical Reasoning

### STANDARD 3

**The student uses mathematical reasoning.**

### PRE-ALGEBRA GRADES 7 OR 8

**To meet this standard, the student will:**

**Benchmark: 7/8.3.1 Analyze information**

**Indicators:**

- Interpret, compare, and contrast information from a variety of sources
- Validate thinking and mathematical ideas using models, known facts, patterns, relationships, and counter-examples

**Benchmark: 7/8.3.2 Predict results and make inferences**

**Indicators:**

- Make and explain conjectures and inferences based on analysis of problem solving

**Benchmark: 7/8.3.3 Draw conclusions and verify results**

**Indicators:**

- Test conjectures and inferences by formulating a proof or by constructing a counterexample
- Verify results and explain why they are accurate or not
- Support arguments and justify results using logical reasoning
- Check for reasonableness of results
- Reflect on and evaluate procedures and results and make necessary revisions

# Mathematical Communication

## STANDARD 4

The student communicates knowledge and understanding in both everyday and mathematical language.

### PRE-ALGEBRA GRADES 7 OR 8

To meet this standard, the student will:

#### Benchmark: 7/8.4.1 Gather information

##### Indicators:

- Develop a plan for collecting information
- Use reading, listening, and observation skills to access and extract mathematical information from multiple sources such as pictures, diagrams, physical models, oral narratives, and symbolic representations
- Use available technology to browse and retrieve mathematical information from a variety of sources

#### Benchmark: 7/8.4.2 Organize and interpret information

##### Indicators:

- Organize and clarify mathematical information by reflecting, verbalizing, discussing, or writing

#### Benchmark: 7/8.4.3 Represent and share information

##### Indicators:

- Clearly and effectively express mathematical ideas with clarity using both everyday and mathematical language appropriate to audience
- Express mathematical ideas to teachers, parents, peers and younger students in appropriate ways.

# Mathematical Connections

## STANDARD 5

The student understands how mathematical ideas connect to other subject areas and real-life situations

## PRE-ALGEBRA GRADES 7 OR 8

To meet this standard, the student will:

**Benchmark: 7/8.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Connect conceptual and procedural understandings among different mathematical content areas
- Relate and use different mathematical models and representations for the same situation

**Benchmark: 7/8.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Identify mathematical patterns and ideas in other disciplines
- Use mathematical thinking and modeling in other disciplines
- Understand scientific notation

**Benchmark: 7/8.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Identify situations in which mathematics can be used to solve problems with local, national, or international implications (such as calculating resources necessary for interstate highway maintenance)
- Investigate the use of mathematics within several occupational/career areas of interest

# Concepts and Procedures of Mathematics

### STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

MIDDLE SCHOOL AND HIGH SCHOOL

## ALGEBRA I, GRADES 8 AND 9

To meet this standard, the student will:

**Benchmark: MS/HS.1.1 Number and Numeration. Understand and apply concepts and procedures from number sense**

**Indicators:**

- Understand operations on real numbers
- Understand order of operations on real numbers and algebraic expressions
- Recognize the order of inequalities of real numbers
- Understand rational and irrational numbers
- Understand how to use the axioms of the real number system
- Understand and use properties and symbolic representations of real numbers
- Understand and apply the concepts of ratio and proportion
- Classify, order, and use operations on real numbers
- Understand concepts of and use processes involving prime and composite numbers, factors and multiples, and divisibility

**Benchmark: MS/HS.1.2 Computation**

**Indicators:**

- Use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving real numbers
- Compute with real numbers, powers and roots
- Preliminary introduction to rational square roots
- Know how to simplify radical expressions using addition, subtraction, multiplication, and division
- Know how to solve simple radical equations
- Know how to solve quadratic equations by factoring
- Know how to use quadratic equation to solve application problems
- Know how to use properties of addition and multiplication to simplify expressions
- Know how to find the slope of a line, given two points on the line, an equation of the line, or the graph of the line
- Know how to solve simultaneous equations.

**Benchmark: MS/HS.1.3 Estimation**

**Indicators:**

- Identify situations involving real numbers in which estimation is sufficient and computation is not required
- Use estimation to predict computation results and determine the reasonableness of answers involving real numbers

**Benchmark: MS/HS.1.4 Attributes and Dimensions. Understand and apply concepts and procedures from measurement**

**Indicators:**

- Understand how changes in dimension affect perimeter, area
- Measure objects and events directly or use indirect methods

**Benchmark: MS/HS.1.5 Approximation and Precision**

**Indicators:**

- Understand that the precision and accuracy of measurement is affected by the measurement tools and calculating procedures
- Use estimation to obtain reasonable approximations

**Benchmark: MS/HS.1.6 Systems and Tools**

**Indicators:**

- Use both of the US and metric systems

**Benchmark: MS/HS.1.7 Shape and Dimension. Understand and apply concepts from geometric sense**

**Indicators:**

- Graph lines given equations
- Find the perimeter of a polygon
- Solve word problems involving area and perimeter of polygons

**Benchmark: MS/HS.1.8 Relationships and Transformations**

**Indicators:**

- Understand and know how to graph inequalities on a number line
- Understand and know how to find solution sets of combined inequalities
- Understand and use coordinate grids to identify and graph on a number line and coordinate plane; points, functions, coordinates of open sentences
- Understand and construct know how to graphs of ordered pairs of numbers in a coordinate plane and find the coordinates of any point in the plane
- Understand construct graphs, and know how to graph any linear equation in two variables
- Know how to find an equation of a line, given: 1. Slope and a point, 2. Two points, 3. Slope and y-intercept and explain your reasoning
- Determine if lines are parallel, perpendicular, or neither and explain your reasoning

**Benchmark: MS/HS.1.9 Probability and Statistics. Understand and apply concepts and procedures from probability and statistics**

**Indicators:**

- Understand the properties of dependent and independent events
- Understand theoretical probability of a simple event
- Use both experimental and theoretical methods to determine probabilities
- Compute percentages
- Use statistics to support different point of view

- Organize and display data in appropriate forms
- Calculate and use the different measures of central tendency, variability, and range as appropriate in describing sets of data
- Find the mean, median and mode of a set of data

**Benchmark: MS/HS.1.10 Relations and Representations. Understand and apply concepts and procedures from algebraic sense**

**Indicators:**

- Recognize, create, extend and generalize patterns, sequences and series
- Translate among tabular, symbolic, and graphical representations of relations
- Represent situations that involve variable quantities with expressions, formulas and equations, and inequalities
- Know how to graph sets of real numbers on a number line
- Graph using different methods

**Benchmark: MS/HS.1.11 Operations**

**Indicators:**

- Evaluate and simplify expressions
- Create and solve equations and inequalities
- Know how to add, subtract, multiply, and divide polynomials
- Apply the distributive, commutative, and associative properties
- Understand how to factor numerical and variable monomials and polynomials by using several various methods
- Know how to solve systems of equations by graphing, substitution, or elimination multiplication, or combining
- Simplify monomials and polynomials

# Mathematical Problem Solving

## STANDARD 2

The student uses mathematics to define and solve problems.

MIDDLE SCHOOL AND HIGH SCHOOL

ALGEBRA I, GRADES 8 AND 9

To meet this standard, the student will:

**Benchmark: MS/HS.2.1 Investigate situations**

**Indicators:**

- Search systematically for patterns in word problems
- Analyze and use multiple strategies to investigate situations
- Identify what information is missing or extraneous and compensate for it
- Analyze an unproductive approach and attempt to modify it or try a new approach

**Benchmark: MS/HS.2.2 Formulate questions and define the problem**

**Indicators:**

- Identify questions to be answered in complex situations
- Define problems in complex situations
- Identify the unknowns in complex situations
- Solve algebraic equations
- Determine solutions to systems of equations

**Benchmark: MS/HS.2.3 Construct solutions**

**Indicators:**

- Organize and synthesize information from multiple sources
- Select and use appropriate mathematical tools
- Apply appropriate methods, operations, and processes to construct a solution
- Solve problems by working backwards
- Make a diagram to solve problems

# Mathematical Reasoning

**STANDARD 3**

**The student uses mathematical reasoning.**

**MIDDLE SCHOOL AND HIGH SCHOOL**

**ALGEBRA I, GRADES 8 AND 9**

**To meet this standard, the student will:**

**Benchmark: MS/HS.3.1 Analyze situations**

**Indicators:**

- Interpret and integrate, compare and contrast information from a variety of sources
- Validate thinking and mathematical ideas using models, known facts, patterns, relationships, counter-examples, and proportional reasoning (justify solutions)

**Benchmark: MS/HS.3.2 Predict results and make inferences**

**Indicators:**

- Understand direct and indirect variation
- Make and explain conjectures and inferences based on analysis of problem situations

**Benchmark: MS/HS.3.3 Draw conclusions and verify results**

**Indicators:**

- Test conjectures and inferences by formulating a proof or by constructing a counterexample
- Support arguments and justify results using inductive and deductive reasoning
- Solve problems by working backwards
- Verify results and explain why they are true or false
- Reflect on and evaluate procedures and results and make necessary revisions changes
- Check for reasonableness of results

# Mathematical Communication

## STANDARD 4

**The student communicates knowledge and understanding in both everyday and mathematical language.**

**MIDDLE SCHOOL AND HIGH SCHOOL**

**ALGEBRA I, GRADES 8 AND 9**

**To meet this standard, the student will:**

**Benchmark: MS/HS.4.1 Gather information**

**Indicators:**

- Develop or select an efficient system for collecting information
- Use reading, listening, and observation skills to access and extract mathematical information from multiple, self-selected sources
- Integrate the use of a variety of available technologies to browse, select, and retrieve mathematical information from multiple sources
- Use information to solve algebraic problems
- Develop a plan for collecting information

**Benchmark: MS/HS.4.2 Organize and interpret information**

**Indicators:**

- Organize, clarify, and refine mathematical information in multiple ways - reflecting, verbalizing, discussing, or writing
- Calculate percent change
- Relate equalities and inequalities

**Benchmark: MS/HS.4.3 Represent and share information**

**Indicators:**

- Graph functions and relations
- Clearly and effectively express complex mathematical ideas using both everyday and mathematical language and notation appropriate to the audience

# Mathematical Connections

## STANDARD 5

**The student understands how mathematical ideas connect to other subject areas and real-life situations**

**MIDDLE SCHOOL AND HIGH SCHOOL**

**ALGEBRA I, GRADES 8 AND 9**

**To meet this standard, the student will:**

**Benchmark: MS/HS.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Relate and connect conceptual and procedural understandings among different mathematical content areas
- Relate and use various mathematical models and representations for the same situation

**Benchmark: MS/HS.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Extend mathematical thinking and modeling in other disciplines
- Describe examples of contributions to the development of mathematics
- Extend mathematical patterns and ideas to other disciplines
- Translate verbal expressions into equations and formulas

**Benchmark: MS/HS.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Identify situations in which mathematics can be used to solve problems with local, national, or international implications
- Investigate the mathematical knowledge and training requirements for occupational/career areas of interest
- Know how to solve word problems using systems of equations
- Understand and apply percentages to real life situations

# Concepts and Procedures of Mathematics

**STANDARD 1**  
**The student understands and applies the concepts and procedures of mathematics.**

## GEOMETRY

### HIGH SCHOOL, GRADES 9 AND 10

**To meet this standard, the student will:**

**Benchmark: HS.1.1    Number and Numeration. Understand and apply concepts and procedures from number sense**

**Indicators:**

- Explain the magnitude of numbers by comparing and ordering real numbers
- Understand and apply the concepts of ratio and both direct and indirect proportion
- State, use and apply properties from Algebra
- State and use theorems and corollaries for the similarity and ratio of perimeters of regular polygons

**Benchmark: HS.1.2    Computation**

**Indicators:**

- Apply the angle sum and exterior angle theorems for triangles
- Find the perimeters of polygons
- Find the measure of an interior or exterior angles of a polygon
- Find the measure of inscribed and central angles
- State and use theorems to find the measures of angles formed by tangents and chords of a circle, two tangents, a secant and a tangent or two chords
- Understand the concept of area, and how to compute it for polygons and circles
- State and use the Pythagorean theorem and its converse
- State and use the theorems and corollaries relating to special right triangles
- Find lateral areas, surface areas, and volumes of specific polyhedron and spheres

- Use the distance formula for computing the distance between two points in a coordinate plane
- Use trigonometry to solve right triangles
- Use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving real numbers and graphing

**Benchmark: HS.1.3 Estimation**

**Indicators:**

- Identify situations involving real numbers in which estimation is sufficient and computation is not required
- Use estimation to predict computation results and to determine the reasonableness of answers involving real numbers

**Benchmark: HS.1.4 Attributes and Dimensions. Understand and apply concepts and procedures from measurement**

**Indicators:**

- Understand how changes in dimension affect perimeter, area, and volume
- Identify several algebraic and geometric symbols
- Understand the interaction of points, lines and planes
- Identify congruent segments
- Identify angles formed by two parallel lines and a transversal
- Understand the properties of angles
- Define, name, and classify an angle
- State the properties of angle congruence
- Understand the properties of triangles and their relationships to each other
- Identify parts of and classify triangles by sides and angles
- Understand the properties of parallel and perpendicular lines and planes
- Recognize the relationship of perpendicular and parallel lines and planes
- Understand the inequality theorems
- Identify and name polygons and their parts, and describe relationships of the parts.
- Find the perimeters of polygons
- Find the measure of an interior or exterior angle of a polygon
- Understand properties of quadrilaterals (rectangle, square, rhombus and trapezoid)
- Name and classify parallelograms
- Identify and use theorems about the diagonals of parallelograms
- Identify and name trapezoids and their associated parts
- Prove and apply theorems about isosceles trapezoids and median of a trapezoid
- Understand the properties of angles, area, chords, tangents and secants to solve problems involving circles
- Find the area of a sector of a circle
- Understand circles and their parts
- Find the measures of area and angles in circles
- Define, name and identify polyhedron
- Describe the relationships between the areas and the lengths of the sides of similar polygons

- Understand the relationship between regular polygons and circles
- Name, classify and identify the parts of angles, polygons, circles, triangles, polyhedron and spheres

**Benchmark: HS.1.5 Approximation and Precision**

**Indicators:**

- Understand that the precision and accuracy of measurement is affected by the measurement tools and calculating procedures
- Use estimation to obtain reasonable approximations
- Find the slope of a line
- Find the distance between points, lines, and planes
- Understand the triangle inequality theorems
- State and use the median theorem of a trapezoid
- Find the measures of arcs and angles in circles
- State and use the Pythagorean theorem and its converse
- Use the theorems and corollaries relating to special right triangles
- State and use the AA, SAS and SSS similarity theorems
- Calculate area for polygons and circles
- Find the perimeter and circumference of polygons and circles
- Prove that two triangles are congruent by using SSS, SAS, ASA, HL, and AAS
- Apply inequality properties to determine whether a triangle exists and order sides and angles

**Benchmark: HS.1.6 Systems and Tools**

**Indicators:**

- Understand the benefits of standard units of measurement and the advantages of the metric system
- Compare, contrast, and use both the US and metric systems
- Select and use tools that will provide an appropriate degree of precision
- Use protractors to measure a given angle
- Use a compass and straightedge to create geometric constructions

**Benchmark: HS.1.7 Shape and Dimension. Understand and apply concepts from geometric sense**

**Indicators:**

- Understand Euclidean constructions
- Compare, describe and classify 2- and 3-dimensional geometric figures
- Construct geometric models and scale drawings using tools as appropriate
- Name and classify polygons, circles, polyhedron and spheres

**Benchmark: HS.1.8 Relationships and Transformations**

**Indicators:**

- Understand and use coordinate grids
- Understand and use properties of symmetry, similarity, and congruence

- Investigate and identify congruency and similarity relationships
- Use a variety of tools and technologies to perform geometric constructions
- Identify the relationship between points, lines, and planes
- Identify the name of segment, a segment midpoint, a ray and opposite ray
- State, illustrate, and use postulates and theorems about point, line and plane relationships
- Identify and use angle relationships
- Identify right angles and perpendicular lines
- Classify the different types of triangles
- Recognize corresponding parts of congruent and similar triangles
- Describe the ways that two lines and planes are related
- Recognize the relationship of perpendicular and parallel lines and planes
- Understand and use the inequality theorems
- Find the measure of an interior or exterior angle of a polygon or their sum
- Identify and use theorems about the diagonals of parallelograms
- State and use the median theorem of a trapezoid
- Understand circles and their properties with chords, secants and tangents
- Understand arcs and angles and their relationships with circles

**Benchmark: HS.1.9 Probability and Statistics. Understand and apply concepts and procedures from probability and statistics**

**Indicators:**

- Be able to calculate the geometric mean of two numbers

**Benchmark: HS.1.10 Relations and Representations. Understand and apply concepts and procedures from algebraic sense**

**Indicators:**

- Recognize, create, extend, and generalize patterns, sequences, and series
- Understand, develop, and express rules describing patterns
- Represent situations that involve variable quantities with expressions, formulas and equations, and inequalities
- Identify the relation between equations and their graphs

**Benchmark: HS.1.11 Operations**

**Indicators:**

- Evaluate and simplify expressions
- Create and solve equations and inequalities
- State and use the properties of congruent statements of segments and polygons
- Use congruent triangles to prove angles and segments congruent
- Calculate an angle from the intercepted arcs
- Apply the inequality theorems
- Find the perimeters of polygons
- State and use the Pythagorean theorem and its converse
- State and use the theorems, corollaries, and properties relating to special right triangles

- Find the area of triangles, polygons, and circles
- Find the surface area and volume of solids
- Simplify radical expressions in order to use trigonometry
- Use the distance formula to find the distance between two points
- Use trigonometric ratios to solve right triangles
- State and use basic right triangle congruency theorems
- Use proportions and ratios to find the length of sides of similar triangles
- Solve systems of linear equations by more than one method

# Mathematical Problem Solving

## STANDARD 2

The student uses mathematics to define and solve problems.

## GEOMETRY

### HIGH SCHOOL, GRADES 9 AND 10

To meet this standard, the student will:

**Benchmark: HS.2.1 Investigate situations**

**Indicators:**

- Search systematically for patterns in order to solve problems or to develop proofs
- Analyze and use multiple strategies
- Identify what information is missing or extraneous and compensate for it
- Analyze an unproductive approach and attempt to modify it or try a new approach
- Use deductive reasoning to write a hypothesis
- Draw accurate models of two and three-dimensional figures

**Benchmark: HS.2.2 Formulate questions and define the problem**

**Indicators:**

- Identify questions to be answered in a problem or proof
- Define problems in complex situations
- Identify the unknowns in complex situations
- Develop an organized plan for a proof

**Benchmark: HS.2.3 Construct solutions**

**Indicators:**

- Organize and synthesize information from multiple sources

- Select and use appropriate mathematical tools
- Apply appropriate methods, operations, and processes to construct a solution
- Use appropriate theorems, postulates and definitions to develop a proof

# Mathematical Reasoning

## STANDARD 3

The student uses mathematical reasoning.

## GEOMETRY

### HIGH SCHOOL, GRADES 9 AND 10

To meet this standard, the student will:

**Benchmark: HS.3.1 Analyze Information**

**Indicators:**

- Interpret and integrate information from multiple sources
- Validate thinking and mathematical ideas using models, known facts, patterns, relationships, counter-examples, and proportional reasoning
- Use diagrams and tables to help discover patterns
- Recognize and use conditional statements
- Use the laws of logic to reason and reach the answer to a problem
- Recognize and use postulates that are stated as conditional statements
- Understand concepts of hypothesis, conclusion and counter example
- Identify the name of segment, a segment midpoint, a ray and opposite rays
- State and use the properties of congruent statements
- State, illustrate, and use postulates and theorems about point, line and plane relationships
- Define and state the properties of congruent angles and angle bisector
- Define supplementary, complementary, adjacent, vertical angles and linear pair
- Describe the ways that lines and/or planes are related and the resulting properties and postulates
- Make converse and bi-conditional statements from conditional statements

**Benchmark: HS.3.2 Predict results and make inferences**

**Indicators:**

- Make and explain conjectures and inferences based on analysis of problem situations
- Use properties from algebra
- Use properties of congruence.

- Write an argument and algorithm using given facts and properties
- Use deductive reasoning to verify angle relationships

**Benchmark: HS.3.3 Draw conclusions and verify results**

**Indicators:**

- Test conjectures and inferences by formulating a proof or by constructing a counterexample
- Support arguments and justify results using inductive and deductive reasoning
- Check for reasonableness of results
- Reflect on and evaluate procedures and results and make necessary revisions and give counterexample
- Give examples and identify deductive and inductive reasoning
- Solve problems by alternate methods such as paragraph proofs, two-column or flow proofs
- Use indirect reasoning to prove an inequality

# Mathematical Communication

## STANDARD 4

The student communicates knowledge and understanding in both everyday and mathematical language.

## GEOMETRY

### HIGH SCHOOL, GRADES 9 AND 10

To meet this standard, the student will:

**Benchmark: HS.4.1 Gather information**

**Indicators:**

- Develop or select an efficient system for collecting information
- Using reading, listening, and observation skills to access and extract mathematical information from multiple, self-selected sources
- Identify several algebraic and geometric symbols
- Give examples and identify deductive and inductive reasoning
- Organize and analyze given information in a proof

**Benchmark: HS.4.2 Organize and interpret information**

**Indicators:**

- Organize, clarify, and refine mathematical information in multiple ways - reflecting, verbalizing, discussing, or writing a plan
- Give examples and identify examples of deductive and inductive reasoning

**Benchmark: HS.4.3 Represent and share information**

**Indicators:**

- Understand the basic vocabulary and terminology of geometry
- Express complex geometric ideas and situations using mathematical language and notation in appropriate and efficient forms
- Express or present mathematical ideas clearly and effectively using both everyday and mathematical language appropriate to audience
- Give examples and identify deductive and inductive reasoning
- Verbally explain a proof and its plan (using diagrams)

## Mathematical Connections

### STANDARD 5

**The student understands how mathematical ideas connect to other subject areas and real-life situations**

### GEOMETRY

### HIGH SCHOOL, GRADES 9 AND 10

**To meet this standard, the student will:**

**Benchmark: HS.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Relate and use conceptual and procedural understandings among multiple mathematical content areas such as Algebra and Trigonometry
- Relate and use multiple equivalent mathematical models and representations
- Relate and use trigonometry to solve geometric problems
- Give examples and identify deductive and inductive reasoning
- Use trigonometry to find parts of parts of right triangles
- Use trigonometry with angles of elevation and depression
- Use Pythagorean theorem to find measurement or distance
- Use properties of quadrilaterals to solve practical problems

**Benchmark: HS.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Extend mathematical patterns and ideas to other disciplines
- Apply mathematical thinking and modeling in other disciplines
- Describe examples of contributions to the development of mathematics

- Use ruler and compass for technical geometric constructions
- Relate geometric properties to other scientific formulas
- Use algebra to solve for missing measure
- Use algebraic manipulation on geometric formulas

**Benchmark: HS.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Identify situations in which mathematics can be used to solve problems with local, national, or international implications
- Investigate the mathematical knowledge and training requirements for occupational/career areas of interest
- Use apply trigonometry with angles of elevation and depression
- Use the Pythagorean theorem and other geometry properties to find necessary measurements of constructions
- Use trigonometric and geometric properties to find measurements

# Concepts and Procedures of Mathematics

**STANDARD 1**  
**The student understands and applies the concepts and procedures of mathematics.**

## ALGEBRA II AND TRIGONOMETRY

### HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.1.1 Number Sense and Numeration. Understand and apply concepts and procedures from number sense**

**Indicators:**

- Explain the magnitude of numbers by comparing and ordering real and complex numbers
- Understanding concepts of and use processes involving prime and composite numbers, factors and multiples, and divisibility
- Understand and apply the concepts of ratio and both direct and indirect proportion
- Understand and determine the subsets of real numbers
- Understand and use imaginary numbers

**Benchmark: HS.1.2 Computation**

**Indicators:**

- Use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving real number
- Compute the inverse of a trig value
- Compute the distance between 2 points
- Compute real roots of a number
- Using substitution and elimination to determine the solution of a linear system
- Expand on adding, subtracting, multiplying, and dividing radicals
- Compute discriminants
- Compute (by hand and by calculator) any of the six trigonometric values of an angle

**Benchmark: HS.1.3 Estimation****Indicators:**

- Identify situations involving real numbers in which estimation is sufficient and computation is not required
- Use estimation to predict computation results and to determine the reasonableness of answers involving real numbers
- Approximate the solution of a linear system
- Estimate the radian measure of an angle

**Benchmark: HS.1.4 Attributes and Dimensions. Understand and apply concepts and procedures from****Indicators:**

- Understand how changes in dimensions affect perimeter, area, and volume
- Measure objects and events directly or use indirect methods
- Calculate rate and other derived and indirect measurements
- Determine dimension of a graph

**Benchmark: HS.1.5 Approximation and Precision****Indicators:**

- Understand that the precision and accuracy of measurement is affected by the measurement tools and calculating procedures
- Use estimation to obtain reasonable approximations
- Approximate distance between 2 points

**Benchmark: HS.1.6 Systems and Tools****Indicators:**

- Select and use tools that will provide an appropriate degree of precision

**Benchmark: HS.1.7 Shape and Dimension. Understand and apply concepts from geometric sense****Indicators:**

- Graph in two dimensions
- Determine the amplitude, period and phase shift of a graph

**Benchmark: HS.1.8 Relations and Transformation**

**Indicators:**

- Understand and use coordinate grids

**Benchmark: HS.1.9 Probability and Statistics. Understand and apply concepts and procedures from probability and statistics**

**Indicators:**

- Understand the properties of dependent and independent events
- Understand and use appropriate counting procedures to determine probabilities
- Use both experimental and theoretical methods to determine probabilities
- Use statistics to support different point of view
- Collect data using appropriate methods and technology
- Organize and display data in appropriate forms
- Calculate and use the different measures of central tendency, variability, and range as appropriate in describing sets of data

**Benchmark: HS.1.10 Relations and Representations. Understand and apply concepts and procedures from algebraic sense**

**Indicators:**

- Recognize, create, extend, and generalize patterns, sequences, and series
- Understand, develop, and express rules describing patterns
- Translate among tabular, symbolic, and graphical representations of relations
- Represent situations that involve variable quantities with expressions, formulas and equations, and inequalities
- Understand and determine the difference between relations and functions
- Understand the concept of domain and range
- Understand the relationship between imaginary and real numbers
- Determine if a graph is a cosine or sine wave

**Benchmark: HS.1.11 Operations**

**Indicators:**

- Evaluate and simplify complex expressions
- Create and solve equations and inequalities
- Extend the use of exponents
- Simplify monomial and polynomial expressions, using all four mathematical operations
- Simplify square roots of variable expressions
- Simplify radical expressions using addition, subtraction, multiplication and addition
- Multiply polynomials containing radicals\
- Solve equations with radicals

# Mathematical Problem Solving

## STANDARD 2

The student uses mathematics to define and solve problems.

## ALGEBRA II AND TRIGONOMETRY

### HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.2.1 Investigate situations**

**Indicators:**

- Design and conduct experiments to verify or disprove predictions
- Understand and make inferences based on the analysis of experimental results
- Search systematically for patterns in complex situations
- Analyze and use multiple strategies
- Identify what information is missing or extraneous and compensate for it
- Analyze an unproductive approach and attempt to modify it or try a new approach
- Determine if a trinomial factors
- Determine if a binomial factors
- Determine if a trigonometric statement is true or false

**Benchmark: HS.2.2 Formulate Questions**

**Indicators:**

- Identify questions to be answered in complex situations
- Define problems in complex situations
- Identify the unknowns in complex situations

**Benchmark: HS.2.3 Constructs Solutions**

**Indicators:**

- Organize and synthesize information from multiple sources
- Select and use appropriate mathematical tools
- Apply appropriate methods, operations, and processes to construct a solution
- Solve algebraic equations for one variable
- Solve algebraic inequalities (and graph solutions)
- Solve equations involving radicals
- Solve trigonometric equations

# Mathematical Reasoning

## STANDARD 3

The student uses mathematical reasoning.

## ALGEBRA II AND TRIGONOMETRY

### HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.3.1 Analyze Information**

**Indicators:**

- Interpret and integrate information from multiple sources
- Validate thinking and mathematical ideas using models, known facts, patterns, relationships, counter-examples, and proportional reasoning
- Given information regarding a linear function, write the equation of the line

**Benchmark: HS.3.2 Predicts results and makes inferences**

**Indicators:**

- Make and explain conjectures and inferences based on analysis of problem situations
- Determine if two lines are parallel or perpendicular
- Predict and determine solutions to algebraic quadratic equations by graphing

**Benchmark: HS.3.3 Draws conclusions and verifies results**

**Indicators:**

- Test conjectures and inferences by formulating a proof or by constructing a counterexample
- Support arguments and justify results using inductive and deductive reasoning
- Check for reasonableness of results
- Reflect on and evaluate procedures and results and make necessary revisions
- Be able to check algebraic solutions
- Determine if a number is a solution to a quadratic equation

# Mathematical Communication

## STANDARD 4

The student communicates knowledge and understanding in both everyday and mathematical language.

## ALGEBRA II AND TRIGONOMETRY

### HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.4.1 Gathers information**

**Indicators:**

- Develop or select an efficient system for collecting information
- Using reading, listening, and observation skills to access and extract mathematical information from multiple, self-selected sources
- Integrate the use of a variety of available technologies to browse, select, and retrieve mathematical information from multiple sources
- Using constraints given in a problem, create a method to solve it

**Benchmark: HS.4.2 Interprets information**

**Indicators:**

- Organize, clarify, and refine mathematical information in multiple ways – reflecting, verbalizing, discussing, or writing
- Organize linear constraints by interpreting information given in a story problem

**Benchmark: HS.4.3 Represents and shares information**

**Indicators:**

- Express complex ideas and situations using mathematical language and notation in appropriate and efficient forms
- Express or present mathematical ideas clearly and effectively using both everyday and mathematical language appropriate to audience

## Mathematical Connections

### STANDARD 5

The student understands how mathematical ideas connect to other subject areas and real-life situations

## ALGEBRA II AND TRIGONOMETRY

## HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.5.1**      **Relates within mathematics**

**Indicators:**

- Relate and use conceptual and procedural understandings among multiple mathematical content areas
- Relate and use multiple equivalent mathematical models and representations

**Benchmark: HS.5.2**      **Relates in Real-life Situations**

**Indicators:**

- Extend mathematical patterns and ideas to other disciplines
- Apply mathematical thinking and modeling in other disciplines
- Describe examples of contributions to the development of mathematics
- Identify situations in which mathematics can be used to solve problems with local, national, or international implications
- Investigate the mathematical knowledge and training requirements for occupational/career areas of interest
- Apply a graph to real-life situations

# Concepts and Procedures of Mathematics

## STANDARD 1

**The student understands and applies the concepts and procedures of mathematics.**

## PRE-CALCULUS

## HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.1.1**      **Number and Numeration. Understand and apply concepts and procedures from number sense**

**Indicators:**

- Understand rational and irrational numbers
- Understand complex numbers, and can write them in polar form
- Understand and use the number “e” in computations

**Benchmark: HS.1.2 Computation**

**Indicators:**

- Compute the  $n^{\text{th}}$  term of sequence
- Compute distance and midpoint between points
- Compute slope of a line
- Compute the sum of  $n^{\text{th}}$  term of sequence
- Compute the sum of a summation
- Compute the critical points
- Compute the zeros of a function
- Compute solutions to interest related problems
- Compute the 6 trigonometric functions of special angles
- Compute the reference and co-terminal of an angle
- Compute coordinates of points
- Compute discriminate
- Compute the vertex of a parabola

**Benchmark: HS.1.3 Estimation**

**Indicators:**

- Estimate slope
- Estimate the polar coordinates, given the rectangular coordinates
- Identify situations involving real numbers in which estimation is sufficient and computation is not required
- Estimate the six trigonometric functions of an angle
- Estimate the sum of a series

**Benchmark: HS.1.4 Attributes and Dimensions. Understand and apply concepts and procedures from measurement**

**Indicators:**

- Use the distance formula to find the measure of a line
- Understand how changes in a quadratic equation affect the parabola

**Benchmark: HS.1.5 Approximation and Precision**

**Indicators:**

- Approximate the sum of a series
- Approximate the roots of polynomial equations
- Approximate the sides, angles and areas of oblique triangles

**Benchmark: HS.1.6 Systems and Tools**

**Indicators:**

- Graph complex type polynomial functions
- Find roots of polynomials

- Find points of intersections of two or more graphed functions
- Effectively use functions keys to find values of trigonometric, functions, e, logarithms, and others pertinent to calculus
- Overall continue to develop mastery and speed in the use of a graphing calculator

**Benchmark: HS.1.7    Shape and Dimension. Understand and apply concepts from geometric sense**

**Indicators:**

- Graph piecewise functions
- Understand the concept of asymptotes and be able to determine vertical asymptotes of rational functions
- Graph polynomial equations and inequalities
- Determine types of discontinuity
- Determine if a graph is symmetrical
- Given a shape, identify the type of conic section

**Benchmark: HS.1.8    Relationships and Transformation**

**Indicators:**

- Use transformations of basic parent graphs
- Determine if a function has an inverse

**Benchmark: HS.1.9    Probability and Statistics. Understand and apply concepts and procedures from probability and statistics**

**Indicators:**

- Understand independent and dependent events
- Understand and use appropriate counting procedures to determine probabilities
- Determine probability and odds of a specific outcome
- Understand the concept of mutually exclusive and inclusive events
- Use permutations and combinations
- Predict statistical outcomes
- Determine if a statistical problem is a permutation or combination

**Benchmark: HS.1.10    Relations and Representations. Understand and apply concepts and procedures from algebraic sense**

**Indicators:**

- Determine if a relation is a function
- Determine the domain and range of a relation
- Determine if a relation has an inverse
- Determine the characteristics of polynomial and rational functions to sketch the graph of the function
- Determine if a sequence is arithmetic or geometric

**Benchmark: HS.1.11    Operations**

**Indicators:**

- Use operations with complex numbers
- Use operations with complex numbers in polar form
- Evaluate and simplify expressions using rational exponents

# Mathematical Problem Solving

**STANDARD 2**

**The student uses mathematics to define and solve problems.**

**PRE-CALCULUS**

**HIGH SCHOOL**

**To meet this standard, the student will:**

**Benchmark: HS.2.1 Investigate situations**

**Indicators:**

- Use situational investigations to practice linear prediction
- Use situational investigations to practice modeling linear programming

**Benchmark: HS.2.2 Formulate questions and define the problem**

**Indicators:**

- Write systems of linear equations given certain information
- Determine solutions to systems of equations

**Benchmark: HS.2.3 Construct solutions**

**Indicators:**

- Solve systems of non-linear equations
- Solve logarithmic and exponential equations
- Solve trigonometric equations

# Mathematical Reasoning

**STANDARD 3**

**The student uses mathematical reasoning.**

## PRE-CALCULUS

### HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.3.1 Analyze Information**

**Indicators:**

- Use definitions and theorems to determine the nature of roots and factors of polynomials
- Use and understand math induction
- Understand and formulate sigma notation

**Benchmark: HS.3.2 Predict results and make inferences**

**Indicators:**

- Predict the relationship between variables from information about the degree of the polynomial

**Benchmark: HS.3.3 Draw conclusions and verify results**

**Indicators:**

- Verify roots by substitution after deriving from theorem
- Verify shape of relations via graphing

# Mathematical Communication

## STANDARD 4

The student communicates knowledge and understanding in both everyday and mathematical language.

## PRE-CALCULUS

### HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.4.1 Gather information**

**Indicators:**

- Develop a plan for collecting information
- Analyze and evaluate the mathematical thinking and strategies of others

**Benchmark: HS.4.2 Organize and interpret information**

**Indicators:**

- Calculate probabilities
- Organize, and clarify information for other students

**Benchmark: HS.4.3 Represent and share information**

**Indicators:**

- Graph Functions relations
- Clearly and effectively express mathematical ideas with clarity using mathematical language and notation

## Mathematical Connections

### STANDARD 5

The student understands how mathematical ideas connect to other subject areas and real-life situations

### PRE-CALCULUS

### HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Determine how various graphs are related
- Use common logarithms to solve exponential problems
- Use the law of sines and cosines to solve triangles

**Benchmark: HS.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Extend mathematical thinking and modeling in other disciplines
- Describe examples of contributions to the development of mathematics
- Extend mathematical patterns and ideas to other disciplines

**Benchmark: HS.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Solve interest problems using given formulas

- Use interest problems to represent real life situations
- Represent problems to real life situations

# Concepts and Procedures of Mathematics

## STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

## CALCULUS

## HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.1.1    Number and Numeration. Understand and apply concepts and procedures from number sense**

**Indicators:**

- Understand and use all types of numbers
- Easily uses all the axioms and properties for real numbers

**Benchmark: HS.1.2    Computation**

**Indicators:**

- Compute slope of a line using derivatives
- Compute slope of a tangent line using information from derivatives
- Determine limits using algebra
- Compute the numerical derivative of a function
- Compute the value of a definite integral
- Compute the maximum or minimum of a function using the function itself
- Computation of Riemann sums using left, right, and midpoint evaluation points
- Numerically solve definite integrals (both manually and by using the appropriate calculator functions)
- Compute solutions for the constant in antiderivatives to solve initial value problems
- Compute area and volume using the Fundamental Theorem of Calculus and/or other rules of integration

**Benchmark: HS.1.3    Estimation**

**Indicators:**

- Estimate the value of a function at a given point
- Subjectively evaluate the accuracy of answers to numerical problems
- Estimate limits from graphs or tables of data

**Benchmark: HS.1.4 Attributes and Dimensions. Understand and apply concepts and procedures from measurement**

**Indicators:**

- Construct mental images of two and three dimensional representations of functions
- Determine slopes of graphs both algebraically and graphically using calculus

**Benchmark: HS.1.5 Approximation and Precision**

**Indicators:**

- Approximate points of intersection and see implications for calculus
- Approximate roots of equations and see implications for calculus

**Benchmark: HS.1.6 Systems and Tools**

**Indicators:**

- Use a graphical calculator to: (all in addition to basic uses of calculators in algebra and precalculus)
  - a. Plot the graph of a function within an arbitrary viewing window
  - b. Find the zeros of a function (solve equations numerically)
  - c. Numerically calculate the derivative of a function
  - d. Numerically calculate the value of a definite integral

**Benchmark: HS.1.7 Shape and Dimension. Understand and apply concepts from geometric sense**

**Indicators:**

- Graph and interpret graphs in 2 and 3 dimensions as a beginning to applying calculus
- Recognize the type of graph from its shape
- Define asymptotes using end behavior models
- Sketch graphs from analytic information using calculus
- Mentally visualize shapes of graphs created from rotations
- Know the geometric implications of graphs of continuous and discontinuous functions
- Understand the corresponding characteristics of the graphs of a function, its first derivative, and its second derivative
- Understand the relationship between the concavity of a function and the sign of its second derivative
- Understand that points of inflection are places where concavity changes
- Visualize a definite integral as a limit of Riemann sums over equal subdivisions

**Benchmark: HS.1.8 Relationships and Transformations**

**Indicators:**

- Use rules of transformations to graph functions
- Transform parent graphs using basic shifts
- See relationships between trigonometric functions
- See relationships between functions and their inverses

- Interpret graphs of first and second derivative functions

**Benchmark: HS.1.9 Probability and Statistics. Understand and apply concepts and procedures from probability and statistics**

**Not applicable**

**Benchmark: HS.1.10 Relations and Representations. Understand and apply concepts and procedures from algebraic sense**

**Indicators:**

- Be able to determine the domain and range of complex functions
- Understand the connection between a derivative and the secant line of the function
- Understand the relationship between the increasing and decreasing behavior of the function and the sign of the function's first derivative
- Set up an integral to represent an area or volume

**Benchmark: HS.1.11 Operations**

**Indicators:**

- Find the equation for a tangent of a line derivative functions
- Use operations of limits to evaluate the limit of a function
- Compute derivatives of basic functions, including power, exponential, logarithmic, and trigonometric
- Know and use basic rules for the derivative of sums, products, and quotients of functions
- Know how to apply the chain rule and implicit differentiation
- Solve initial value problems

# Mathematical Problem Solving

## STANDARD 2

The student uses mathematics to define and solve problems.

## CALCULUS

## HIGH SCHOOL

**To meet this standard, the student will:**

**Benchmark: HS.2.1 Investigate situations**

**Indicators:**

- Determine maximums and minimums for geometric objects using analytic techniques
- Solve a variety of optimization problems

- Investigate and construct related rates and optimization problems

**Benchmark: HS.2.2 Formulate questions and define the problem**

**Indicators:**

- Use Riemann sums to find area under a curve
- Model rates of change, including related rates problems
- Solve optimization problems
- Find the average value of a function

**Benchmark: HS.2.3 Construct solutions**

**Indicators:**

- Interpret derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Determine if a graph has a limit, a maximum or minimum value
- Use the Fundamental Theorem to evaluate definite integrals
- Solve for volume, and surface area of a solid

# Mathematical Reasoning

## STANDARD 3

The student uses mathematical reasoning.

## CALCULUS

## HIGH SCHOOL

**To meet this standard, the student will:**

**Benchmark: HS.3.1 Analyze Information**

**Indicators:**

- Determine if given information is vital to solving the problem

**Benchmark: HS.3.2 Predict results and make inferences**

**Indicators:**

- Have intuitive understanding of continuity
- Predict the limit of a function (if any)
- Make inferences about a solution, after finding the maximum and minimum

**Benchmark: HS.3.3 Draw conclusions and verify results**

**Indicators:**

- Check answers for sensibility
- Understand continuity in terms of limits
- Use derivatives to verify correct integration

# Mathematical Communication

## STANDARD 4

**The student communicates knowledge and understanding in both everyday and mathematical language.**

## CALCULUS

## HIGH SCHOOL

**To meet this standard, the student will:**

**Benchmark: HS.4.1 Gather information**

**Indicators:**

- Given a picture, formulate equations that represent the graph(s)
- Infer first and second derivatives from graphs of functions
- Make inferences from graphs of functions that represent derivatives

**Benchmark: HS.4.2 Organize and interpret information**

**Indicators:**

- Sketch graphs using the analytic information gathered from calculating the first and second derivatives
- Interpret physical representations of derivatives
- Understand interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration

**Benchmark: HS.4.3 Represent and share information**

**Indicators:**

- Verbally present solutions to problems and conclusions reached using poster presentations
- Verbal descriptions are translated into equations involving derivatives and vice versa
- Verbally share logic and reasoning used to solve problems
- Practice vocabulary of the calculus

# Mathematical Connections

## STANDARD 5

The student understands how mathematical ideas connect to other subject areas and real-life situations

## CALCULUS

## HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.5.1** Relate concepts and procedures within mathematics

**Indicators:**

- Understand when radian measures of an angles must be used
- Use trigonometric identities to simplify calculus problems
- Use various techniques for evaluating limits
- Use various rules to find a derivative of the same function

**Benchmark: HS.5.2** Relate mathematical concepts and procedures to other disciplines

**Indicators:**

- Use derivatives to understand and solve velocity and other rate of change problems related to physics

**Benchmark: HS.5.3** Relate mathematical concepts and procedures to real-life situations

**Indicators:**

- Understand the derivative as representing an instantaneous rate of change
- Solve rate of change problems
- Solve related rates problems
- Set up velocity problems using derivatives
- Use integration to solve velocity problems

# Concepts and Procedures of Mathematics

## STANDARD 1

The student understands and applies the concepts and procedures of mathematics.

# ADVANCED PLACEMENT CALCULUS

## HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.1.1 Number and Numeration. Understand and apply concepts and procedures from number sense**

**Indicators:**

- Understand and use all types of numbers
- Easily uses all the axioms and properties for real numbers

**Benchmark: HS.1.2 Computation**

**Indicators:**

- Compute slope of a line using derivatives
- Compute slope of a tangent line using information from derivatives
- Determine limits using algebra
- Compute the numerical derivative of a function
- Compute the value of a definite integral
- Compute the maximum or minimum of a function using the function itself
- Compute the maximum or minimum of a function using the Mean Value Theorem
- Computation of Riemann sums using left, right, and midpoint evaluation points
- Numerically solve definite integrals (both manually and by using the appropriate calculator function)
- Compute solutions for the constant in antiderivatives to solve initial value problems
- Compute area and volume using the Fundamental Theorem of Calculus and/or other rules of integration

**Benchmark: HS.1.3 Estimation**

**Indicators:**

- Estimate the value of a function at a given point
- Subjectively evaluate the accuracy of answers to numerical problems
- Estimate limits from graphs or tables of data

**Benchmark: HS.1.4 Attributes and Dimensions. Understand and apply concepts and procedures from measurement**

**Indicators:**

- Construct mental images of two and three dimensional representations of functions
- Determine slopes of graphs both algebraically and graphically using calculus

**Benchmark: HS.1.5 Approximation and Precision**

**Indicators:**

- Approximate points of intersection and see implications for calculus
- Approximate roots of equations and see implications for calculus

**Benchmark: HS.1.6 Systems and Tools**

**Indicators:**

- Use a graphical calculator to: (all in addition to basic uses of calculators in algebra and precalculus)
  - e. Plot the graph of a function within an arbitrary viewing window
  - f. Find the zeros of a function (solve equations numerically)
  - g. Numerically calculate the derivative of a function
  - h. Numerically calculate the value of a definite integral

**Benchmark: HS.1.7 Shape and Dimension. Understand and apply concepts from geometric sense**

**Indicators:**

- Graph and interpret graphs in 2 and 3 dimensions as a beginning to applying calculus
- Recognize the type of graph from its shape
- Define asymptotes using end behavior models
- Sketch graphs from analytic information using calculus
- Mentally visualize shapes of graphs created from rotations
- Understand graphical implications of even and odd functions
- Understand the geometric implications of graphs of continuous and discontinuous functions
- Understand the geometric implications of the Intermediate Value Theorem
- Understand the corresponding characteristics of the graphs of a function, its first derivative, and its second derivative
- Understand the relationship between the concavity of a function and the sign of its second derivative
- Understand that points of inflection are places where concavity changes
- Visualize a definite integral as a limit of Riemann sums over equal subdivisions

**Benchmark: HS.1.8 Relationships and Transformations**

**Indicators:**

- Use rules of transformations to graph functions
- Transform parent graphs using basic shifts
- See relationships between trigonometric functions
- See relationships between functions and their inverses
- Interpret graphs of first and second derivative functions

**Benchmark: HS.1.9 Probability and Statistics. Understand and apply concepts and procedures from probability and statistics.**

**Not applicable**

**Benchmark: HS.1.10 Relations and Representations. Understand and apply concepts and procedures from algebraic sense**

**Indicators:**

- Be able to determine the domain and range of complex functions
- Understand the connection between a derivative and the secant line of the function
- Understand the relationship between the increasing and decreasing behavior of the function and the sign of the function's first derivative
- Set up an integral to represent an area or volume

**Benchmark: HS.1.11 Operations**

**Indicators:**

- Find the equation for a tangent of a line using derivative functions
- Use operations of limits to evaluate the limit of a function
- Compute derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Know and use basic rules for the derivative of sums, products, and quotients of functions
- Know how to apply the chain rule and implicit differentiation
- Solve initial value problems

# Mathematical Problem Solving

**STANDARD 2**

**The student uses mathematics to define and solve problems.**

## ADVANCED PLACEMENT CALCULUS

### HIGH SCHOOL

**To meet this standard, the student will:**

**Benchmark: HS.2.1 Investigate situations**

**Indicators:**

- Determine maximums and minimums for geometric objects using analytic techniques
- Solve a variety of optimization problems
- Investigate related rates and optimization problems

**Benchmark: HS.2.2 Formulate questions and define the problem**

**Indicators:**

- Use Riemann sums to find area under a curve
- Model rates of change, including related rates problems
- Solve optimization problems
- Find the average value of a function

**Benchmark: HS.2.3 Construct solutions**

**Indicators:**

- Interpret derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Determine if a graph has a limit
- Determine if a graph has a maximum or minimum value
- Use the Fundamental Theorem to evaluate definite integrals
- Use the Fundamental Theorem to represent a particular antiderivative, and for analytical and graphical analysis of functions so defined
- Solve for volume, and surface area of a solid

# Mathematical Reasoning

## STANDARD 3

**The student uses mathematical reasoning.**

**HIGH SCHOOL**

**To meet this standard, the student will:**

**Benchmark: HS.3.1 Analyze Information**

**Indicators:**

- Determine if given information is vital to solving the problem

**Benchmark: HS.3.2 Predict results and make inferences**

**Indicators:**

- Have intuitive understanding of continuity
- Predict the limit of a function (if any)
- Make inferences about a solution, after finding the maximum and minimum

**Benchmark: HS.3.3 Draw conclusions and verify results**

**Indicators:**

- Check answers for sensibility
- Understand continuity in terms of limits
- Use derivatives to verify correct integration

# Mathematical Communication

## STANDARD 4

The student communicates knowledge and understanding in both everyday and mathematical language.

### ADVANCED PLACEMENT CALCULUS

#### HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.4.1 Gather information**

**Indicators:**

- Given a picture, formulate equations that represent the graph(s)
- Infer first and second derivatives from graphs of functions
- Make inferences from graphs of functions that represent derivatives

**Benchmark: HS.4.2 Organize and interpret information**

**Indicators:**

- Sketch graphs using the analytic information gathered from calculating the first and second derivatives
- Interpret physical representations of derivatives
- Understand interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration

**Benchmark: HS.4.3 Represent and share information**

**Indicators:**

- Verbally present solutions to problems and conclusions reached
- Verbal descriptions are translated into equations involving derivatives and vice versa
- Verbally share logic and reasoning used to solve problems
- Practice vocabulary of the calculus

# Mathematical Connections

## STANDARD 5

The student understands how mathematical ideas connect to other subject areas and real-life situations

# ADVANCED PLACEMENT CALCULUS

## HIGH SCHOOL

To meet this standard, the student will:

**Benchmark: HS.5.1 Relate concepts and procedures within mathematics**

**Indicators:**

- Understand when radian measures of an angles must be used
- Use trigonometric identities to simplify calculus problems
- Use various techniques for evaluating limits
- Use various rules to find a derivative of the same function
- Relate the fundamental theorem of calculus to mean value theorem for integrals
- Understand the relationship between the Shell and Disc methods used to find volumes

**Benchmark: HS.5.2 Relate mathematical concepts and procedures to other disciplines**

**Indicators:**

- Use derivatives to understand and solve velocity and other rate of change problems related to physics
- Use derivatives to understand and solve problems related to biology (e.g. bacteria growth), chemistry (e.g. carbon depletion, and economics (e.g. marginal profit)

**Benchmark: HS.5.3 Relate mathematical concepts and procedures to real-life situations**

**Indicators:**

- Understand the derivative as representing an instantaneous rate of change
- Solve rate of change problems
- Solve related rates problems
- Set up velocity problems using derivatives
- Use integration to solve velocity problems
- Determine the amount of growth or decay using an exponential model

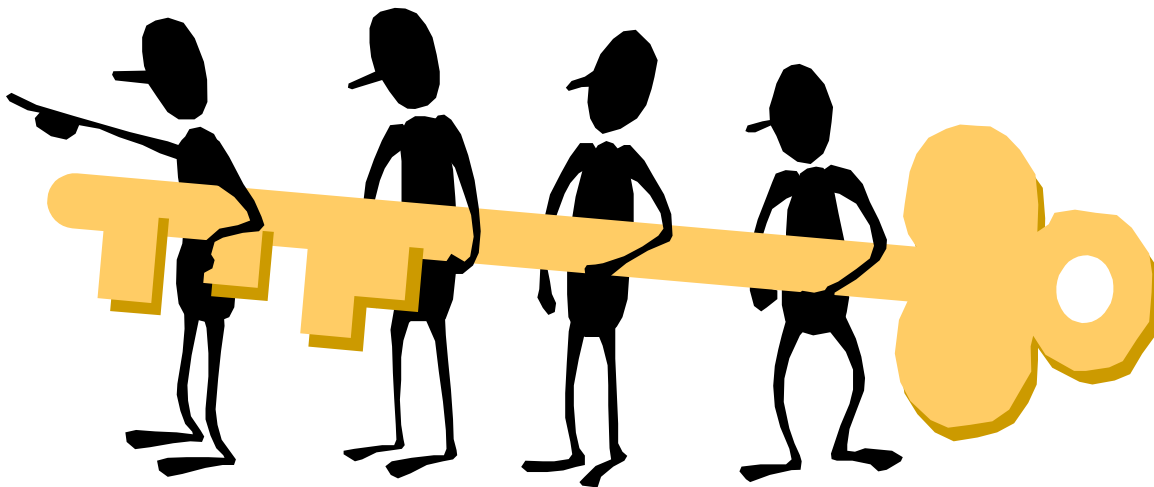
**Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. ~NCTM Standards, 2000**

## **ASSESSMENT**

The assessment of students' understanding includes the use of multiple forms of assessment such as portfolios, rubrics, math journals, teacher-made tests, and group projects. However students and parents, and possibly some teachers, may find these forms unfamiliar. If ISD teachers and administrators are committed to putting the NCTM assessment principles into action, it will take time. Teachers will need in-service training specifically aimed at developing their assessment techniques and administrators will need to support teachers in helping students and parents understand the purpose of such approaches. This is a big undertaking, but it is also one that the Mathematics Committee is willing to embrace.

The pages in the assessment section of this curriculum will include some examples of good assessment practices, as well as information on the major shifts in assessment. We invite the teachers at ISD to include their ideas to the assessment section so we can all benefit from your expertise.

## **THE KEY**



**TO SUCCEED IS WORKING AND  
LEARNING TOGETHER.**